Organizing Committee:

Jean-Philippe Noël, University of Liège
Maarten Schoukens, Vrije Universiteit Brussel
Gaëtan Kerschen, University of Liège
Johan Schoukens, Vrije Universiteit Brussel

Scientific Committee:

Bart Peeters, Siemens
Ebrahim Louarroudi, Universiteit Antwerpen
Carl Edward Rasmussen, University of Cambridge
Thomas Schön, Uppsala Universitet
Steve Vanlanduit, Universiteit Antwerpen
Keith Worden, The University of Sheffield

Organized with the support of:
## PROGRAM

### MONDAY 24/04/2017

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
<th>Speaker(s)</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>08.30 – 09.00</td>
<td>Registration &amp; Coffee</td>
<td></td>
<td></td>
</tr>
<tr>
<td>09.00 – 09.30</td>
<td>Welcome &amp; Introduction</td>
<td>J.P. Noël &amp; M. Schoukens</td>
<td></td>
</tr>
<tr>
<td>09.30 – 10.30</td>
<td>Keynote 1</td>
<td>B. Peeters – LMS: Siemens PLM Software</td>
<td>Structural non-linearities – an industrial view</td>
</tr>
<tr>
<td>10.30 – 11.00</td>
<td>Coffee</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.00 – 12.30</td>
<td>Session 1</td>
<td>T. Dossogne, J.P. Noël and G. Kerschen</td>
<td>Nonlinear system identification of an F-16 aircraft using the</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>acceleration surface method</td>
</tr>
<tr>
<td></td>
<td></td>
<td>K. Tiels – Polynomial nonlinear state-space modeling of the</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>F-16 aircraft benchmark</td>
</tr>
<tr>
<td></td>
<td></td>
<td>P. Dreesen, K. Tiels and M. Ishteva – Decoupling</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>nonlinear models for the F-16 ground vibration test benchmark</td>
</tr>
<tr>
<td>12.30 – 13.45</td>
<td>Lunch</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13.45 – 14.45</td>
<td>Keynote 2</td>
<td>C. Rasmussen and J. Schoukens</td>
<td>Bayesians methods in system identification: equivalences, differences,</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>and misunderstanding</td>
</tr>
<tr>
<td>14.45 – 15.15</td>
<td>Session 2</td>
<td>G. Hollander, P. Dreesen, M. Ishteva and J. Schoukens</td>
<td>Nonlinear model decoupling using a tensor decomposition initialization</td>
</tr>
<tr>
<td>15.15 – 15.45</td>
<td>Coffee</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15.45 – 16.45</td>
<td>Session 3</td>
<td>T. Münker, T.O. Heinz and O. Nelles – Regularized local</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>FIR model networks for a Bouc-Wen and a Wiener-Hammerstein system</td>
</tr>
<tr>
<td></td>
<td></td>
<td>E. Zhang and M. Schoukens – Fast location of process</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>noise for nonlinear system identification</td>
</tr>
<tr>
<td>16.45 – 17.15</td>
<td>Discussion Session 1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
# TUESDAY 25/04/2017

<table>
<thead>
<tr>
<th>Time</th>
<th>Session</th>
<th>Speaker(s)</th>
<th>Topic/Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>09.00 – 10.00</td>
<td>Keynote 3</td>
<td>L. Ljung – Linkoping University</td>
<td>Non-linear system identification: A palette from off-white to pit-black</td>
</tr>
<tr>
<td>10.00 – 10.30</td>
<td>Session 4</td>
<td>B. Tang – On the interaction of an electro-dynamic shaker and a beam with stiffness nonlinearity</td>
<td></td>
</tr>
<tr>
<td>10.30 – 11.00</td>
<td>Coffee</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>M. Rébillat and M. Schoukens – M. Schoukens</td>
<td>A methodology to compare two estimation methods for parallel Hammerstein models</td>
</tr>
<tr>
<td>12.30 – 13.45</td>
<td>Lunch</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17.00 – 19.00</td>
<td>Francqui Inaugural Lecture</td>
<td>G. Kerschen – University of Liège</td>
<td>G. Kerschen – University of Liège Nature is nonlinear. What about engineering structures?</td>
</tr>
<tr>
<td>17.00 – 19.00</td>
<td>Francqui Inaugural Lecture</td>
<td>G. Kerschen – University of Liège</td>
<td>G. Kerschen – University of Liège Nature is nonlinear. What about engineering structures?</td>
</tr>
</tbody>
</table>
# WEDNESDAY 26/04/2017

<table>
<thead>
<tr>
<th>Time</th>
<th>Session/Event</th>
<th>Speaker(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>09.00 – 10.00</td>
<td>Keynote 4</td>
<td>D. Barton – Bristol University</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Control-based continuation - from models to experiments</td>
</tr>
<tr>
<td>10.00 – 10.30</td>
<td>Session 7</td>
<td>S.R. Hassan – System identification of dynamic force transducers</td>
</tr>
<tr>
<td>10.30 – 11.00</td>
<td>Coffee</td>
<td></td>
</tr>
<tr>
<td>11.00 – 12.00</td>
<td>Session 8</td>
<td>A.F. Esfahani, P. Dreesen, J.P. Noël, K. Tiels and J. Schoukens</td>
</tr>
<tr>
<td></td>
<td></td>
<td>– Decoupled polynomial nonlinear state space models of a Bouc-Wen hysteretic system</td>
</tr>
<tr>
<td></td>
<td></td>
<td>D. Westwick, G. Hollander and J. Schoukens – The decoupled polynomial NARX model: parameter reduction and structural insights for the Bouc-Wen benchmark</td>
</tr>
<tr>
<td>12.00 – 12.30</td>
<td>Discussion Session 3 &amp; Closing</td>
<td>J.P. Noël &amp; M. Schoukens</td>
</tr>
<tr>
<td>12.30 – 13.45</td>
<td>Lunch</td>
<td></td>
</tr>
</tbody>
</table>
Benchmark Setups
1 Introduction

Hysteresis is a phenomenology commonly encountered in very diverse engineering and science disciplines, ranging from solid mechanics, electromagnetism and aerodynamics [1, 2, 3] to biology, ecology and psychology [4, 5, 6]. The defining property of a hysteretic system is the persistence of an input-output loop as the input frequency approaches zero [7]. Hysteretic systems are inherently nonlinear, as the quasi-static existence of a loop requires an input-output phase shift different from 0 and 180 degrees, which are the only two options offered by linear theory. The root cause of hysteresis is multistability [8]. A hysteretic system possesses multiple stable equilibria, attracting the output depending on the input history. In this sense, it is appropriate to refer hysteresis as system nonlinear memory.

This document describes the synthesis of noisy data exhibiting hysteresis behaviour carried out by combining the Bouc-Wen differential equations (Section 2) and the Newmark integration rules (Section 3). User guidelines to an accurate simulation are provided in Section 4. The test signals and the figures of merit that are used in this benchmark are presented in Section 5. Anticipated nonlinear system identification challenges associated with the present benchmark are listed in Section 6.
2 The Bouc-Wen model of hysteresis

The Bouc-Wen model [9, 10] has been intensively exploited during the last decades to represent hysteretic effects in mechanical engineering, especially in the case of random vibrations. Extensive literature reviews about phenomenological and applied aspects related to Bouc-Wen modelling can be found in Refs. [11, 12].

The vibrations of a single-degree-of-freedom Bouc-Wen system, i.e. a Bouc-Wen oscillator with a single mass, is governed by Newton’s law of dynamics written in the form [10]

\[ m_L \ddot{y}(t) + r(y, \dot{y}) + z(y, \dot{y}) = u(t), \]  

(1)

where \( m_L \) is the mass constant, \( y \) the displacement, \( u \) the external force, and where an over-dot indicates a derivative with respect to the time variable \( t \). The total restoring force in the system is composed of a static nonlinear term \( r(y, \dot{y}) \), which only depends on the instantaneous values of the displacement \( y(t) \) and velocity \( \dot{y}(t) \), and of a dynamic, i.e. history-dependent, nonlinear term \( z(y, \dot{y}) \), which encodes the hysteretic memory of the system. In the present study, the static restoring force contribution is assumed to be linear, that is

\[ r(y, \dot{y}) = k_L y + c_L \dot{y}, \]  

(2)

where \( k_L \) and \( c_L \) are the linear stiffness and viscous damping coefficients, respectively. The hysteretic force \( z(y, \dot{y}) \) obeys the first-order differential equation

\[ \dot{z}(y, \dot{y}) = \alpha \dot{y} - \beta \left( \gamma |\dot{y}| |z|^{\nu-1} z + \delta |\dot{y}|^\nu \right), \]  

(3)

where the five Bouc-Wen parameters \( \alpha, \beta, \gamma, \delta \) and \( \nu \) are used to tune the shape and the smoothness of the system hysteresis loop. Table 1 lists the values of the physical parameters selected in this study. The linear modal parameters deduced from \( m_L, c_L \) and \( k_L \) are given in Table 2. Fig. 1 (a) illustrates the existence of a non-degenerate loop in the system input-output plane for quasi-static forcing conditions. In comparison, by setting the \( \beta \) parameter to 0, a linear behaviour is retrieved in Fig. 1 (b). The excitation \( u(t) \) in these two figures is a sine wave with a frequency of 1 Hz and an amplitude of 120 N. The response exhibits no initial condition transients as it is depicted over 10 cycles in steady state.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( m_L )</th>
<th>( c_L )</th>
<th>( k_L )</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( \gamma )</th>
<th>( \delta )</th>
<th>( \nu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value (in SI unit)</td>
<td>2</td>
<td>10</td>
<td>5 ( \times 10^4 )</td>
<td>5 ( \times 10^4 )</td>
<td>1 ( \times 10^3 )</td>
<td>0.8</td>
<td>-1.1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1: Physical parameters of the Bouc-Wen system.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Natural frequency ( \omega_0 ) (Hz)</th>
<th>Damping ratio ( \zeta ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>35.59</td>
<td>1.12</td>
</tr>
</tbody>
</table>

Table 2: Linear modal parameters of the Bouc-Wen system.
Figure 1: Hysteresis loop in the system input-output plane for quasi-static forcing conditions. (a) Non-degenerate loop obtained for the parameters in Table 1; (b) linear behaviour retrieved when setting the $\beta$ parameter to 0.

### 3 Time integration

The Bouc-Wen dynamics in Eqs. (1) and (3) can be effectively integrated in time using a Newmark method. Newmark integration relies on one-step-ahead approximations of the velocity and displacement fields obtained by applying Taylor expansion and numerical quadrature techniques [13]. Denoting by $h$ the integration time step, these approximation relations write

\[
\begin{align*}
\dot{y}(t + h) &= \dot{y}(t) + (1 - a) \; h \; \ddot{y}(t) + a \; h \; \ddot{y}(t + h) \\
y(t + h) &= y(t) + h \; \dot{y}(t) + \left( \frac{1}{2} - b \right) \; h^2 \; \ddot{y}(t) + b \; h \; \ddot{y}(t + h).
\end{align*}
\]  

(4)

Parameters $a$ and $b$ are typically set to 0.5 and 0.25, respectively. Eqs. (4) are herein enriched with an integration formula for the variable $z(t)$, which takes the form

\[
z(t + h) = z(t) + (1 - c) \; h \; \dot{z}(t) + c \; h \; \ddot{z}(t + h),
\]

(5)

where $c$, similarly to $a$, is set to 0.5. Based on Eqs. (4) and (5), a Newmark scheme proceeds in two steps. First, predictions of $\dot{y}(t + h)$, $y(t + h)$ and $z(t + h)$ are calculated assuming $\ddot{y}(t + h) = 0$ and $\ddot{z}(t + h) = 0$. Second, the initial predictors are corrected via Newton-Raphson iterations so as to satisfy the dynamic equilibria in Eqs. (1) and (3).

### 4 User guidelines to an accurate simulation

The Newmark integration of the Bouc-Wen dynamics in Eqs. (1) and (3) is implemented in the Matlab encrypted p-file BoucWen_NewmarkIntegration.p. This function features 5 inputs, namely:
• the integration time step $h$;
• the external force time history $u(t)$;
• the initial value of the displacement $y(t = 0)$;
• the initial value of the velocity $\dot{y}(t = 0)$;
• the initial value of the hysteretic force $z(t = 0)$.

The single output of the function is the displacement time history $y(t)$.

Based on the authors’ experience with the Newmark integration of the Bouc-Wen system of Section 2, the following guidelines are formulated:

• it is suggested to consider a working sampling frequency of 750 Hz in order to properly observe the harmonic components generated by the nonlinearity;
• it is strongly advised to upsample the input force $u(t)$ by a factor 20 during time integration to guarantee the accuracy of the resulting displacement time series. This comes down to setting the integration sampling frequency, i.e. $1/h$, to 15000 Hz;
• after integration, the output sequence $y(t)$ may be low-pass filtered and downsampled using the Matlab command `decimate`. Note that this command belongs to the Matlab Signal Processing toolbox;
• low-pass filtering may be achieved using a 30-th order FIR filter (argument ‘fir’ of the `decimate` command), paying attention to the inherent edge effects of the filter;
• the `decimate` command may be called several times breaking the downsampling argument, e.g. 20, into its prime factors, e.g. $2 \cdot 2 \cdot 5$, to enhance numerical precision;
• initial conditions on $y(t)$, $\dot{y}(t)$ and $z(t)$ are usually set to 0.

The minimal working example file `BoucWen_ExampleIntegration.m` implements all these guidelines. In this example, a multisine excitation [14] is applied to the Bouc-Wen system considering all excited frequencies in the $5 - 150$ Hz band and a frequency resolution $f_0 = f_s/N \cong 0.09$ Hz, given a sampling frequency $f_s = 750$ Hz and a number of time samples $N = 8192$. The root-mean-squared amplitude of the input is 50 $N$ and 5 output periods are simulated. The sampling rate during integration is set to 15000 Hz. The synthesised displacement time history is low-pass filtered and downsampled back to 750 Hz.

In more details:

• the working and integration sampling frequencies are defined in section `Time integration parameters` on line 10;
the excitation signal is designed in section **Excitation signal design** on line 17. Note that the Newmark simulation algorithm supports, in principle, all types of input time series;

- initial conditions are set on lines 43, 44 and 45;

- time integration is run on line 48;

- low-pass filtering and downsampling are carried out in section **Low-pass filtering and downsampling** on line 51;

- the edge effects of the low-pass filter are addressed by adding an extra period during time integration (see lines 28 and 29) and removing it afterwards (see lines 63, 64 and 65).

Note that Gaussian noise band-limited in 0 – 375 Hz is automatically added to the synthesised measurement of \( y(t) \) considering a root-mean-squared amplitude of \( 8 \times 10^{-3} \) mm. This provides a realistic signal-to-noise ratio of about 40 dB at 50 N excitation level. The input time series \( u(t) \) is assumed to be noiseless.

Fig. 2 (a) displays the calculated system output. The exponential decay of the system transient response is plotted in Fig. 2 (b) by subtracting the last synthesised period from the entire time record. This graph indicates that transients due to initial conditions only affect the first period of measurement, and that the applied periodic input results in a periodic output. It also demonstrates the high accuracy of the Newmark integration, as the transient response reaches the Matlab precision of -313 dB in steady state. Remark that, in this particular case, no noise was added to the output to focus on integration accuracy.

### 5 Model test and figure of merit

Two fixed test datasets are provided through the benchmark meeting website: a random phase multisine and a sine-sweep signal. The test datasets are noiseless and a sampling frequency of 750 Hz is considered. The random phase multisine dataset contains one steady-state period of 8192 samples. The excited band encompasses all frequencies in 5 – 150 Hz, and the RMS input value is 50 N. The sine-sweep dataset is not in steady state, the simulation started with initial conditions equal to zero. In this case, the amplitude of the input is 40 N, and the frequency band from 20 to 50 Hz is covered at a sweep rate of 10 Hz/min. These test sets function as a target for the obtained model, the model should perform as good as possible on these test datasets. The goal of the benchmark is to estimate a good model on the estimation data. The test data should not be used for any purpose during the estimation.

We expect all participants of the benchmark to report the following figure of merit for all
Figure 2: System output calculated in response to a multisine input band-limited in 5 – 150 Hz. (a) Output over 5 periods, with one specific period highlighted in grey; (b) output in logarithmic scaling (in black) and decay of the transient response (in blue).

test datasets to allow for a fair comparison between different methods:

$$e_{\text{RMSt}} = \sqrt{\frac{1}{N_t} \sum_{t=1}^{N_t} (y_{\text{mod}}(t) - y_t(t))^2},$$  \hspace{1cm} (6)

where $y_{\text{mod}}$ is the modeled output, $y_t$ is the output provided in the test dataset, $N_t$ is the total number of points in $y_t$.

Also mention whether the modeled output $y_{\text{mod}}$ is obtained using simulation (only the test input $u_t$ is used to obtain the modeled output $y_{\text{mod}}(t) = F(u_t(1), \ldots, u_t(t)))$ or prediction (both the test input $u_t$ and the past test output $y_t$ are used to obtain the modeled output $y_{\text{mod}}(t) = F(u_t(1), \ldots, u_t(t), y_t(1), \ldots, y_t(t-1)))$. Provide both figures of merit (simulation and prediction) if the identified model allows for it.

6 Nonlinear system identification challenges

We anticipate the Bouc-Wen benchmark to be associated with 4 major nonlinear system identification challenges:

- it possesses a nonlinearity featuring memory, i.e. a dynamic nonlinearity;
- the nonlinearity is governed by an internal variable $z(t)$, which is not measurable;
• the nonlinear functional form in Eq. (3) is nonlinear in the parameter $\nu$;

• the nonlinear functional form in Eq. (3) does not admit a finite Taylor series expansion because of the presence of absolute values.

References


Wiener-Hammerstein benchmark with process noise

M. Schoukens¹, J.P. Noël²

¹ ELEC Department
Vrije Universiteit Brussel, Brussels, Belgium

² Space Structures and Systems Laboratory
Aerospace and Mechanical Engineering Department
University of Liège, Liège, Belgium

1 Introduction

Process noise is already well studied and modeled in the linear time-invariant (LTI) framework. Nonparametric and parametric noise models (Box-Jenkins, ARX, ARMAX) provide good solutions to the LTI process noise problem [1, 2].

Most of the nonlinear modeling approaches only consider additive (colored) noise at the output (see, for instance, the methods listed in [3, 4]), or are restricted to an ARX or ARMAX like noise model (NARX and NARMAX in [5]). Some recent methods consider a more complex noise framework using expectation maximization, particle filter methods, or errors-in-variables approaches [6, 7, 8].

This benchmark presents a Wiener-Hammerstein electronic circuit where the process noise is the dominant noise distortion.

The next sections describe the Wiener-Hammerstein system (Section 2) and describe the data restrictions (Section 3). The test data and the figures of merit that are used in this benchmark are presented in Section 4. Finally, some of the expected challenges during the identification process are listed in Section 5.

2 Wiener-Hammerstein system with process noise

The Wiener-Hammerstein structure is a well known block-oriented system. It contains a static nonlinearity that is sandwiched in between two LTI blocks (Figure 1). The presence of the two LTI blocks results in a problem that is harder to identify. The system is quite similar to the Wiener-Hammerstein system that is studied in an earlier benchmark [9, 10], the main difference is the presence of the process noise.
Figure 1: A Wiener-Hammerstein system with process noise. The LTI blocks at the input and the output are depicted by $R(s)$ and $S(s)$ respectively. $f(x)$ denotes the static nonlinearity. The process noise $e_x(t)$ enters the system before the static nonlinearity. Two smaller (neglectable) noise sources $e_u(t)$ and $e_y(t)$ are present in the measurement channels. $u_m(t)$ and $y_m(t)$ are the measured input and output signals.

The first filter $R(s)$ can be described well with a third order lowpass filter. The second LTI subsystem $S(s)$ is designed as an inverse Chebyshev filter with a stopband attenuation of 40 dB and a cutoff frequency of 5 kHz. The second LTI subsystem has a transmission zero within the excited frequency range. This makes the inversion of $S(s)$ difficult.

The static nonlinearity $f(x)$ is realized with a diode-resistor network, this results in a saturation nonlinearity.

The additive process noise $e_x(t)$ is a filtered white Gaussian noise sequence. The filtered noise is generated starting from a discrete-time 3rd order lowpass Butterworth filter followed by a zero-order hold reconstruction and an analog low-pass reconstruction filter with a cut-off frequency of 20 kHz. The noise sources $e_u(t)$ and $e_y(t)$ account for the measurement noise, they can be considered to be white Gaussian noise sources. The dominant noise source is $e_x(t)$, the measurement noise is very small.

3 Data and user guidelines

3.1 Estimation data

The participants are offered the unique opportunity to design the estimation input signals themselves. The measurements are performed at the VUB ELEC department by an experienced user of the measurement setup during 3 different measurement campaigns. The exact dates of these measurement campaigns will be announced to all subscribed participants via e-mail and on the website. All measured input-output data will be offered to the participants to obtain a good model of the system. The possibility for the participants to perform a short measurement campaign at the ELEC department, VUB can be discussed.
• WH Measurement campaign 1: October/November
• WH Measurement campaign 2: January/February
• WH Measurement campaign 3: March

An initial dataset is available on the benchmark meeting website to perform some first analysis and tests on the system. All the data measured by the participants will also be available to all participants through the benchmark meeting website.

A reference signal will be measured during each new measurement campaign to check the reproducibility of the measurements compared to the previous measurements performed on the system.

3.2 Measurement setup

The inputs and the process noise are generated by an arbitrary zero-order hold waveform generator (AWG), the Agilent/HP E1445A, sampling at 78125 Hz. The generated zero-order hold signals are passed through a reconstruction filter (Tektronix Wavetek 432) with a cut-off frequency of 20 kHz. The in- and output signals of the system are measured by the alias protected acquisition channels (Agilent/HP E1430A) sampling at 78125 Hz. The AWG and acquisition cards are synchronized with the AWG clock, and hence the acquisition is phase coherent to the AWG. Leakage errors are hereby easily avoided. Finally, buffers are added between the acquisition cards and the in- and output of the system to avoid that the measurement equipment would distort the measurements.

3.3 User guidelines

The following restrictions apply for the input signals:

• The input signals should be stored in a .mat file,
• The name of the input signal variable is 'input',
• The variable 'input' has the dimension $N \times M$, where $N$ is the number of points in the signals and $M$ is the number of signals that needs to be measured,
• The maximum length of the signal is $N_{max} = 65536$,
• The maximum number of signals in one file is $M_{max} = 100$,
• The amplitude of the signals should be between -4 and 4,
• Note that the sampling frequency is fixed: $f_s = 78125$ Hz.

The measurement file contains a structure 'dataMeas'. This structure has 4 fields:
• r: reference signal, signal loaded into the generator,
• u: measured input signal,
• y: measured output signal,
• fs: the sample frequency.

4 Model test and figure of merit

Two fixed test sets are provided through the benchmark meeting website: a random phase multisine and a sine-sweep signal. Both signals are measured as periodic signals, the datasets contain one steady-state period of the signal. Both measured input signals have an rms value of 0.71 $V_{rms}$, and they excite the frequencies from DC to 15 kHz, DC not included. The sine-sweep signal covers the frequency band from DC to 15 kHz at a sweep rate of 4.29 MHz/min.

These test sets function as a target for the obtained model, the model should perform as good as possible on these test datasets. The goal of the benchmark is to estimate a good model on the estimation data. The test data should not be used for any purpose during the estimation. The test sets are measured in the absence of process noise. The noiseless test sets can be used to evaluate the bias on the estimate since wrong noise assumptions can lead to a biased estimate of the system under test [11].

We expect all participants of the benchmark to report the following figure of merit for all test datasets to allow for a fair comparison between different methods:

$$e_{RMS} = \sqrt{\frac{1}{N_t} \sum_{t=1}^{N_t} (y_{mod}(t) - y(t))^2},$$  (1)

where $y_{mod}$ is the modeled output, $y_t$ is the output provided in the test dataset, $N_t$ is the total number of points in $y_t$.

Also mention whether the modeled output $y_{mod}$ is obtained using simulation (only the test input $u_t$ is used to obtain the modeled output $y_{mod}(t) = F(u_t(1), \ldots, u_t(t))$) or prediction (both the test input $u_t$ and the past test output $y_t$ are used to obtain the modeled output $y_{mod}(t) = F(u_t(1), \ldots, u_t(t), y_t(1), \ldots, y_t(t-1))$). Provide both figures of merit (simulation and prediction) if the identified model allows for it.

5 Nonlinear system identification challenges

We anticipate the Wiener-Hammerstein benchmark to be associated with 3 major nonlinear system identification challenges:
• the process noise that is present in the system,
• the static nonlinearity which is not directly accessible from neither the measured input or output,
• the output dynamics are difficult to invert due to the presence of a transmission zero.

References

F-16 aircraft benchmark
based on ground vibration test data

J.P. Noël¹, M. Schoukens²

¹ Space Structures and Systems Laboratory
Aerospace and Mechanical Engineering Department
University of Liège, Liège, Belgium

² ELEC Department
Vrije Universiteit Brussel, Brussels, Belgium

1 Introduction and F-16 instrumentation

The experimental data made available to the Workshop participants were acquired on a full-scale F-16 aircraft (see Fig. 1 (a)) on the occasion of the Siemens LMS Ground Vibration Testing Master Class, held in September 2014 at the Saffraanberg military basis, Sint-Truiden, Belgium.

During the test campaign, two dummy payloads were mounted at the wing tips to simulate the mass and inertia properties of real devices typically equipping an F-16 in flight (see Fig. 1 (b)). The aircraft structure was instrumented by means of 145 acceleration sensors. One shaker was attached underneath the right wing to apply input signals (see Fig. 1 (c)). The dominant source of nonlinearity in the structural dynamics was expected to originate from the mounting interfaces of the two payloads. These interfaces consist of T-shaped connecting elements on the payload side, slid through a rail attached to the wing side (see Fig. 1 (d)). A preliminary investigation showed that the back connection of the right-wing-to-payload interface was the predominant source of nonlinear distortions in the aircraft dynamics, and is therefore the focus of this benchmark study.

Measurements were acquired at a sampling frequency of 400 \( Hz \). Two distinct input signals are made available: (1) the voltage measured at the output of the signal generator amplifier, acting as a reference input, and (2) the actual force provided by the shaker and measured by a impedance head at the excitation location. Three acceleration signals are
Figure 1: F-16 instrumentation. (a) Complete aircraft structure; (b) dummy payload mounted at the right wing tip; (c) shaker attached underneath the right wing; (d) back connection of the right-wing-to-payload mounting interface.
provided as output quantities. They were measured (1) at the excitation location, (2) on
the right wing next to the nonlinear interface of interest, and (3) on the payload next to
the same interface. Outputs are listed in this order in the matrices of data.

2 Description of the measured data sets

2.1 Sine-sweep excitation with a linear, negative rate

Sine-sweep excitations with a linear, negative rate of 0.05 \(Hz/s\) (sweep down) were applied
(data files named F16Data_SineSw_Level#.mat). The covered input frequency range was
15 – 2 \(Hz\). Seven different levels of excitation are provided as benchmark data. The
lowest level at 4.8 \(N\) input amplitude can be considered as a linear data set. Three higher
excitation levels are given to function as estimation data in nonlinear regimes of vibration,
namely data sets number 3, 5, 7 corresponding to 28.8, 67.0 and 95.6 \(N\), respectively. Data
sets number 2, 4 and 6 at 19.2, 57.6 and 86.0 \(N\), respectively, are to be used for testing
the models estimated using the data sets 3, 5 and 7, respectively.

2.2 Multisine excitation with a full frequency grid

Data recorded under multisine excitations with a full frequency grid from 2 to 15 \(Hz\)
are provided (data files named F16Data_FullMSine_Level#.mat). At each force level, 9
periods were acquired considering a single realization of the input signal. The number
of points per period is 8192. Note that transients are present in the first period of
measurement. Similarly to the sine-sweep case, seven excitation levels are considered,
starting from linear data at 12.4 \(N\) RMS (data set 1). In addition, three nonlinear
estimation data sets (number 3, 5 and 7 at 36.8, 73.6 and 97.8 \(N\) RMS, respectively) are
accompanied by their corresponding test sets (numbers 2, 4 and 6 at 24.6, 61.4 and 85.7
\(N\) RMS, respectively).

2.3 Multisine excitation with a random frequency grid

Multisines were also applied considering only odd frequencies excited. Moreover, within
each group of 4 successive excited odd lines, 1 frequency line was randomly rejected to act
as a detection line for odd nonlinearities (data files named F16Data_SpecialOddMSine_Level#.mat).
In this setting, the frequency band from 1 to 60 \(Hz\) was excited. 3 periods per level were
recorded, considering 10 input realizations per level. The number of points per period is
16384. Note that only the last 2 periods of each realization are in steady state. The data
sets were originally sampled at 200 \(Hz\). They were upsampled to 400 \(Hz\) in the frequency
domain, processing period per period, and assuming the data is periodic and in steady
state.

Because of the multiple realizations, the number of tested excitation levels was reduced
to 3, namely 12.2, 49.0 and 97.1 N RMS. These 3 levels entail nonlinear oscillations. It is suggested to use, at each level, 9 realizations for estimation and to consider a final realization as test data.

3 Goal of the identification

Identifying a full nonlinear model of the F-16 dynamics represents a great challenge. The goal of this system identification benchmark should therefore be understood in a broader sense, and participants are encouraged to explore other paths for analysis, including:

- general nonparametric analysis of the data.
- linearized modeling.
- linear parameter-varying modeling to track the evolution of the aircraft natural frequencies and damping ratios versus the excitation level.
- nonlinear modeling around a single mode (i.e. a single resonance) of the structure (see Section 5 below).

4 Figure of merit

When a parametric model is estimated, we expect the participants to report the following figure of merit using an appropriate test data set to allow for a fair comparison between different methods:

$$e_{RMS\text{t}} = \sqrt{1/N_t \sum_{t=1}^{N_t} (y_{mod}(t) - y_t(t))^2},$$

(1)

where $y_{mod}$ is the modeled output, $y_t$ is the output provided in the test data set, $N_t$ is the total number of points in $y_t$. This error measure can be evaluated considering a single or multiple outputs.

Also mention whether the modeled output $y_{mod}$ is obtained using simulation (only the validation input $u_t$ is used to obtain the modeled output $y_{mod}(t) = F(u_t(1), \ldots, u_t(t)))$ or prediction (both the validation input $u_t$ and the past validation output $y_t$ are used to obtain the modeled output $y_{mod}(t) = F(u_t(1), \ldots, u_t(t), y_t(1), \ldots, y_t(t-1)))$. Provide both figures of merit (simulation and prediction) if the identified model allows for it.
5 Nonlinear system identification challenges

We anticipate the F-16 benchmark to be associated with 3 major nonlinear system identification challenges:

- the order of the system is reasonably high. In the 2 – 15 Hz band, the F-16 possesses about 10 resonance modes. The first few modes below 5 Hz correspond to rigid-body motions of the structure. The first flexible mode around 5.2 Hz corresponds to wing bending deformations. The mode involving the most substantial nonlinear distortions is the wing torsion mode located around 7.3 Hz. Participants wishing to limit the order of their model should focus on this latter mode.

- the mounting interface of interest is expected to feature nonlinearities in stiffness and damping, due to clearance and friction, respectively.

- clearance and friction may lead to hard nonlinearities, and hence may not be appropriately modeled using smooth basis functions.

Acknowledgments

The authors would like to thank Dr. Bart Peeters, from Siemens PLM Software, for his help and advices during the test campaign.
Cascaded tanks benchmark combining soft and hard nonlinearities

M. Schoukens\textsuperscript{1}, P. Mattsson\textsuperscript{2}, T. Wigren\textsuperscript{2}, J.P. Noël\textsuperscript{3}

\textsuperscript{1} ELEC Department
Vrije Universiteit Brussel, Brussels, Belgium

\textsuperscript{2} Division of Systems and Control
Department of Information Technology
Uppsala University, Uppsala, Sweden

\textsuperscript{3} Space Structures and Systems Laboratory
Aerospace and Mechanical Engineering Department
University of Liège, Liège, Belgium

1 Introduction

Many systems exhibit a quasi linear or weakly nonlinear behavior during normal operation, and a hard saturation effect for high peaks of the input signal. The proposed benchmark is an example of this type of nonlinear system. On top of this, only a short data record is available for the parameter estimation step.

The next sections describe the cascaded tanks system (Section 2) and introduce the estimation and test data (Section 3). The figures of merit that are used in this benchmark are presented in Section 4. Finally, some of the expected challenges during the identification process are listed in Section 5.

2 Cascaded tanks system

The cascaded tanks system is a fluid level control system consisting of two tanks with free outlets fed by a pump. The input signal controls a water pump that pumps the water from a reservoir into the upper water tank. The water of the upper water tank flows through a small opening into the lower water tank, and finally through a small opening from the lower water tank back into the reservoir. This process is shown in Figure 1.

The relation between (1) the water flowing from the upper tank to the lower tank and (2) the water flowing from the lower tank into the reservoir are weakly nonlinear functions.
Figure 1: The cascaded tanks system: the water is pumped from a reservoir in the upper tank, flows to the lower tank and finally flows back into the reservoir. The input is the pump voltage, the output is the water level of the lower tank.

However, when the amplitude of the input signal is too large, an overflow can happen in the upper tank, and with a delay also in the lower tank. When the upper tank overflows, part of the water goes into the lower tank, the rest flows directly into the reservoir. This effect is partly stochastic, hence it acts as an input-dependent process noise source. The overflow saturation nonlinear behavior of the lower tank is clearly visible in the time domain representation of the output signals (see Figure 2). A video of such an overflow situation can be found on the benchmark website.

Without considering the overflow effect, the following input-output model can be constructed based on Bernoulli’s principle and conservation of mass:

\[
\begin{align*}
\dot{x}_1(t) &= -k_1 \sqrt{x_1(t)} + k_4 u(t) + w_1(t), \\
\dot{x}_2(t) &= k_2 \sqrt{x_1(t)} - k_3 \sqrt{x_2(t)} + w_2(t), \\
y(t) &= x_2(t) + e(t),
\end{align*}
\]

where \(u(t)\) is the input signal, \(x_1(t)\) and \(x_2(t)\) are the states of the system, \(w_1(t)\), \(w_2(t)\) and \(e(t)\) are additive noise sources, and \(k_1, k_2, k_3\) and \(k_4\) are constants depending on the system properties.

3 Estimation and test data

The input signals are multisine signals which are 1024 points long, and excite the frequency range from 0 to 0.0144 Hz, both for the estimation and test case. The lowest frequencies have a higher amplitude than the higher frequencies (see Figure 2). The sample period \(T_s\) is equal to 4 s. The input signals are zeroth-order hold input signals.

The process is controlled from a Matlab interface to the A/D and D/A converters attached to the water level sensor and the pump actuator. The water level is measured using
Figure 2: Input (a,c) and output (b,d) signals of the estimation (blue) and test (red) data records in the time (a,b) and frequency (c,d) domain.

capacitive water level sensors, the measured output signals have a signal-to-noise ratio that is close to 40 dB. The water level sensors are considered to be part of the system, they are not calibrated and can introduce an extra source of nonlinear behavior.

Note that the system was not in steady state during the measurements. The system states have an unknown initial value at the start of the measurements. This unknown state is the same for both the estimation and the test data record.

4 Figure of merit

The goal of the benchmark is to estimate a good model on the estimation data. The test data should not be used for any purpose during the estimation.

We expect all participants of the benchmark to report the following figure of merit for all test datasets to allow for a fair comparison between different methods:

\[ e_{RMSt} = \sqrt{\frac{1}{N_t} \sum_{t=1}^{N_t} (y_{mod}(t) - y_t(t))^2}, \]  

where \( y_{mod} \) is the modeled output, \( y_t \) is the output provided in the test data set, \( N_t \) is the total number of points in \( y_t \).
Also mention whether the modeled output $y_{mod}$ is obtained using simulation (only the test input $u_t$ is used to obtain the modeled output $y_{mod}(t) = F(u_t(1), \ldots, u_t(t))$) or prediction (both the test input $u_t$ and the past test output $y_t$ are used to obtain the modeled output $y_{mod}(t) = F(u_t(1), \ldots, u_t(t), y_t(1), \ldots, y_t(t-1))$). Provide both figures of merit (simulation and prediction) if the identified model allows for it.

5 Nonlinear system identification challenges

We anticipate the cascaded tanks benchmark to be associated with 4 major nonlinear system identification challenges:

- the hard saturation nonlinearity combined with the weakly nonlinear behavior of the system in normal operation,
- the overflow from the upper to the lower tank, this effect also introduces input-dependent process noise,
- the relatively short estimation data record,
- the unknown initial values of the states.
Workshop Contributions
Nonlinear System Identification of an F-16 Aircraft Using The Acceleration Surface Method

T. Dossogne
tdossogne@ulg.ac.be

J.P. Noël
jp.noel@ulg.ac.be

G. Kerschen
g.kerschen@ulg.ac.be

Space Structures and Systems Lab, Aerospace and Mechanical Eng. Dept.
Université de Liège, 9 Allée de la Découverte, 4000 Liège, Belgium

1 Framework

In most mechanical engineering applications, the purpose of nonlinear system identification is to upgrade and update a linear finite element (FE) model. Nonlinear elements, whose parameters are found during the identification procedure, are locally added to this linear model to better reproduce the experimental results.

The following nonlinear system identification method takes place in this framework, where a linear FE model of the structure is assumed to be available. A nonlinearity characterization step is first performed using qualitative nonlinear stiffness curves computed with the Acceleration Surface Method (ASM). This step enables to determine the location and the mathematical form of the nonlinear elements to add in the FE model. The parameters of those nonlinear elements are then identified by optimization, using the error between the ASM curves from the experiments and the numerical model as a cost function.

The case of an F-16 aircraft exhibiting nonlinearities at the wing-to-payload mounting interfaces [1] is considered and a FE model of the right wing and its connected payload is used as a starting point.

2 The Acceleration Surface Method

The ASM provides a quick visualization tool of the nonlinearity [2] as it only requires the acceleration, velocity and displacement signals measured on two degrees-of-freedom (DOF) across the studied nonlinear connection and under a sine sweep excitation. Its main assumption consists in approximating the restoring forces acting on a DOF by the opposite of its acceleration. Therefore, acceleration versus relative displacement (resp. velocity) curves yield a qualitative estimation of the stiffness (resp. damping) nonlinearities.

Regarding the F-16 aircraft, Figure 1(a) clearly reveals the presence of a piecewise linear behavior in the connection. Moreover, the discontinuity locations can be directly extracted from the qualitative stiffness curve.

3 Finite Element Model Updating Using the ASM and Optimization

As only qualitative estimation of the nonlinearities can be carried out using the ASM, the FE model is used to compute the actual nonlinear parameters, namely the slopes of the different linear regions in the stiffness curve. Simulations on the FE model with the added nonlinear elements are performed. The ASM is then applied to the simulated results (Figure 1(b)) to produce a qualitative stiffness curve similar to the experimental one. The error between those two stiffness curves is finally used as a cost function for an optimization procedure to identify the real parameters of the nonlinear elements.

References


Figure 1: Qualitative stiffness curves computed by the ASM on (a) experimental results and (b) simulation results
A polynomial nonlinear state-space (PNLSS) model [1] can capture many nonlinear behaviors and is successfully used in a large range of applications. This model is put in modal canonical form (i.e. there are 6 pairs of states that each are involved in modal coupling only within the wing torsion mode and ear interaction is only considered within each resonance, and however, no nonlinear terms in the input are considered, nonlinear interaction is only considered within each resonance, and coupled interaction only within the wing torsion mode and its two neighboring resonances. This reduces the number of parameters in the model from 7826 to 882. The elements of the matrices $A$, $B$, $C$, and $D$, and the free elements of the matrices $E$ and $F$ are optimized by minimizing the mean square model error of the last period of one realization of the estimation set in the focused frequency band.

The PNLSS model slightly improves on the linear model’s prediction of the output spectrum around the wing torsion mode (Fig. 1). The PNLSS model error at the wing torsion mode (at ca. 7.3 Hz). By using a frequency weighting and providing structure in the PNLSS model, it was possible to reduce its complexity.

Future work includes using non-polynomial basis functions that can better capture the expected saturating nonlinearities.
Decoupling Nonlinear Models of the F-16 Aircraft Benchmark

Philippe Dreesen  Koen Tiels  Mariya Ishteva  Johan Schoukens

Vrije Universiteit Brussel (VUB), Dept. VUB-ELEC
firstname.lastname@vub.ac.be

1 From linear to nonlinear models

System identification is witnessing a paradigm shift from linear to nonlinear modeling. Several nonlinear models have been studied, ranging from white-box models that take represent the governing physics, over gray-box model structures where a specific parameterization is considered, to black-box models which contain numerous parameters or may even be nonparametric.

The model classes in the darker shades of gray have more descriptive power than the lighter and structured ones, but they come at a price. They typically have a high parametric complexity, and are more difficult to interpret. For instance, when using polynomials as basis functions, the number of terms of increases combinatorially with the number of variables (i.e., regressors in a NARX model, or states in a state-space model), as with the nonlinear degree.

2 Decoupling nonlinear models

To cope with the increasing complexity, model reduction schemes are indispensable tools for gaining control of the complexity, and hopefully restore insight into the models. In the current research, we will explore a specific model reduction method that works by ‘decoupling’ a nonlinear function in order to reduce its complexity.

We will use the decoupling method [2], which aims at representing a given multivariate nonlinear vector function (i.e., a nonlinear MIMO function) as a sandwiched structure of a set of parallel univariate branches (i.e., SISO functions), surrounded by two linear transformations (Figure 1), formally \( f(u) = Wg(V^T u) \), where \( g(x) = (g_1(x_1), \ldots, g_r(x_r)) \) is a vector containing the \( r \) internal univariate functions. In this way, the originally MIMO nonlinearities are lumped together into simpler SISO nonlinearities. The advantages of decoupling a nonlinear function are two-fold: Decoupling often leads to a drastic reduction of the number of parameters, while possibly providing intuitive insight into the nonlinearities.

The decoupling method proceeds by collecting the Jacobian matrices of the function in a number of sampling points \( u^k \), denoted by \( J(u^k) \). The decomposition can then be obtained by performing a simultaneous matrix diagonalization of the set of matrices \( J(u^k) \), immediately returning the linear transformations, as well as information to construct the internal SISO nonlinearities.

3 Exploring decoupled models for the F-16 benchmark

We will illustrate how the decoupling method can be applied with nonlinear dynamical models of the F-16 aircraft. Specifically, we will explore the use of NARX models [1] and polynomial nonlinear state-space (PNLSS) models [3].

4 Acknowledgements

This work was supported in part by the Fund for Scientific Research (FWO-Vlaanderen), the Flemish Government (Methusalem), the Belgian Government through the Interuniversity Poles of Attraction (IAP VII) Program, the ERC advanced grant SNLSID under contract 320378, and FWO projects G028015N and G090117N.

References

Nonlinear model decoupling using a tensor decomposition initialization

Gabriel Hollander, Philippe Dreesen, Mariya Ishteva, Johan Schoukens
Dept. ELEC, Vrije Universiteit Brussel, Pleinlaan 2, 1050 Brussels, Belgium
gabriel.hollander@vub.ac.be

1 Introduction and problem statement

Finding an approximate model of a given model, by reducing its number of parameters, while keeping the accuracy as good as possible, is of great importance for any existing model. Currently, some nonlinear block-oriented models are hard to interpret, and in need of simplification. For this, we have developed a decoupling method, using tensor decompositions.

Let \( f \) be the multiple-input multiple-output function as shown in Figure 1. It is composed of two transformation matrices \( V \) and \( W \), and a set of single-input single-output functions \( g_1(x_1), \ldots, g_r(x_r) \). Given such a model, we wish to reduce it using a smaller number of branches \( \tilde{r} < r \), which results in reducing the number of parameters, depending on the ratio \( \tilde{r}/r \).

![Figure 1: The multiple-input multiple-output model studied in this abstract.](image)

2 Methodology

A nonlinear and nonconvex cost function, defined on the output of \( f \), is optimized using an iterative algorithm, [1]. A “good” initialization point has to be chosen in order to yield low cost function values after the optimization process. We compared a uniformly randomly chosen initialization with a tensor decomposition initialization, described as follows.

Using the decoupling algorithm described in [2], the given function \( f \) is decoupled using \( \tilde{r} \) branches. The method uses first-order derivative information of \( f \), evaluated in a given number of inputs of \( f \), which are then stacked into a third-order tensor. This tensor can be decomposed using the Canonical Polyadic Decomposition and ultimately yields a decomposed function with \( \tilde{r} \) branches. This is then used as the second kind of initialization for the cost function in [1].

3 Results and conclusion

The proposed tensor decomposition initialization method yields in most cases smaller cost functions than when using a random initialization, Figure 2. The difference is most significant when the ratio \( \tilde{r}/r \) is not too small. The proposed initialization can thus help to reduce the number of parameters of the nonlinear model described in Figure 1. The next step in this research is to reduce more complex models, which use a structured nonlinear multiple-input multiple-output model as \( f \). This could open new model reduction strategies for block-oriented and nonlinear state-space modelling.

![Figure 2: Using the tensor-based initialization, the cost function error is smaller than when using a random initialization point.](image)

4 Acknowledgments

This work was supported in part by the Fund for Scientific Research (FWO-Vlaanderen), the Flemish Government (Methusalem), the Belgian Government through the Interuniversity Poles of Attraction (IAP VII) Program, the ERC advanced grant SNLSID under contract 320378, and FWO projects G028015N and G090117N.

References

Regularized Local FIR Model Networks for a Bouc-Wen and a Wiener-Hammerstein System

Tobias Münker, Tim Oliver Heinz, and Oliver Nelles
University of Siegen, Mechanical Engineering, Automatic Control and Mechatronics
Paul-Bonatz-Str. 9-11, 57076 Siegen
tobias.muenker@uni-siegen.de

1 Approach
The goal of the proposed approach is to provide a universal tool for the identification of nonlinear dynamic systems. This is the reason for evaluating the method for both the Bouc-Wen and the Wiener-Hammerstein system. Some of the results are also described in [1]. Consider that a data set $D = \{u(k), y(k)\}_{k=1}^{N}$ with $u(k)$ denoting the input and $y(k)$ denoting the output at the discrete time step $k$ is given. The goal is to identify the relationship

$$\hat{y}(k) = f(u(k), \ldots, u(k-n), y(k-1), \ldots, y(k-n))$$  \hspace{1cm} (1)

with $f(\cdot)$ denoting a nonlinear function approximator and $n$ denoting the order of the system. Often the consideration which delayed values of $u(k)$ and $y(k)$ to be used as arguments in the $f(\cdot)$ is referred to as dynamic structure identification. Another problem is the choice of an appropriate nonlinear approximator for $f(\cdot)$.

1.1 Local Model Networks
In this contribution local model networks [3] are used, i.e. the approximator is described as

$$\hat{y}(k) = \sum_{i=1}^{m} \left( w_i x(k) + \bar{w}_i \right) \phi_i(z(k))$$  \hspace{1cm} (2)

with the validity functions denoted by $\phi_i(z(k))$, the parameters of the local linear models by $w_i$ and the offset by $\bar{w}_i$. The variables $x(k)$ and $z(k)$ can both contain delayed input as well as output values. Prior knowledge about the linear or nonlinear influence of certain regressors can be used to assign them to $x(k)$ or $z(k)$ respectively. The estimation of the validity functions is done by tree based identification methods as described in [3].

1.2 Regularized FIR Models
The idea of regularized FIR models as described in [2] is to use only delayed values of $u(k)$ for $x(k)$, i.e. to use FIR models as local models and select appropriate delayed values of both delayed input and output values for $z(k)$. The estimation of the FIR coefficients is done as described by [4] with a regularized weighted least square estimate by optimizing

$$\min_{w_i, \bar{w}_i} \sum_{k=1}^{N} \phi_i(z(k)) \left( y(k) - x(k)^T w_i - \bar{w}_i \right)^2 + w_i^T \sigma^2 \mathbf{P}^{-1} w_i$$  \hspace{1cm} (3)

with a kernel matrix $\mathbf{P}$. The regularization term considers both the smoothness and the exponential decay of the impulse response.

2 Results
The test errors evaluated on multisine and sine-sweep data are shown in Fig. 1 for prediction and simulation (index $p$ and $s$) of both the Bouc-Wen and the Wiener-Hammerstein system. Since the local models are FIR models there is no feedback within this part and thus simulation error and prediction error are quite similar. Compared to a linear ARX model the simulation error of the local model network with FIR models is always smaller.

![Figure 1: Figure of merit for two benchmark systems.](image)

References
Fast location of process noise for nonlinear system identification

Erliang Zhang
Department of Mechanical Engineering, Zhengzhou University, 450001 Zhengzhou, China
erliang.zhang@zzu.edu.cn

Maarten Schoukens
Department of ELEC, Vrije Universiteit Brussel, B1050 Brussels, Belgium
maarten.schoukens@vub.ac.be

1 Introduction

The focus on system identification has been changing with more of an emphasis on nonlinear systems over recent years. Nonlinear system identification has been developed by focusing on specific classes of systems, such as block-oriented [1], NARMAX [2] and nonlinear state space systems [3]. The general noise framework which incorporates process noise is one of state-of-the-art challenges in nonlinear system identification. It is non-trivial to locate the process noise w.r.t. the static nonlinearity within the nonlinear system as the disturbance of the process noise is hidden in the measurement noise and nonlinear distortions under general excitations. Attempt has been made in our previous work [4] by developing a measurement protocol which uses a specially designed input that is periodic but non-stationary within one period. This technique is effective, however time-consuming as multiple measurements are needed. The present work contributes to locate the process noise for system modeling by proposing a fast experiment protocol using only one measurement and applying a simple sine signal or narrowband multisine signals. The amplitude of the (multisine)sine signal can be seen as the expected value of the non-stationary input [4].

2 Principal and Example

The present work is illustrated on detecting the process noise for three popular model classes: block-oriented systems, nonlinear state space systems and NARMAX systems. In principal, the second-order property of the non-periodic part of the system response excited by a sine signal will be exploited to reveal the process noise w.r.t. the static nonlinearity. The computed output variance is rearranged by the ordered sine input for the ease of checking the non-stationarity of the output variance. The proposed technique is illustrated on a Wiener-Hammerstein system with a saturation nonlinearity, as seen in Fig. 1. The non-stationarity of the rearranged output variance w.r.t. the ordered sine input tells us that the process noise passes through the static nonlinearity which is the saturation nonlinearity.

3 Conclusion

A user-friendly technique is proposed to locate the process noise for nonlinear system identification using only one experiment. The effectiveness and robustness of the method are justified by simulated examples and the real-life Wiener-Hammerstein benchmark.

Acknowledgment

This work was supported in part by the ERC advanced grant SNLSID 320378, by the Belgian Federal Government (IAP DYSCO VII/19), by the Methusalem grant of the Flemish Government (METH-1), and by the NSF of China (U1504618).

References


Figure 1: Rearranged output variance w.r.t. the ordered input.
On the Interaction of an Electro-dynamic Shaker and a Beam with Stiffness Nonlinearity

Bin Tang
Institute of Internal Combustion Engine, Dalian University of Technology, Dalian, China btang@dlut.edu.cn

Michael J. Brennan
Departamento de Engenharia Mecânica, UNESP, Ilha Solteira, São Paulo, Brasil mjbreannan0@btinternet.com

Gianluca Gatti
Dept. of Mechanical, Energy and Management Eng., University of Calabria, Cosenza, Italy gianluca.gatti@unical.it

1 Introduction

In vibration testing systems, one of the most common exciters is an electro-dynamic shaker [1, 2]. The dynamics of this, coupled with the dynamics of the test structure combine to play an important role in linear and nonlinear parameter estimation. One of the principle problems is that of force “drop-out” close to the resonance frequency of the test structure [1]. Whilst this is not a particular problem for the majority of tests for linear structures, it can be problematic in the testing of nonlinear structures [3], as in many cases it is desirable to maintain a constant excitation force with frequency. The interaction between the electro-dynamic shaker and the nonlinear test structure, needs to be clearly understood so that experimental results can be clearly interpreted and system parameters correctly estimated. This paper describes an experimental study of a particular nonlinear test structure, in the laboratory. The test structure is a compressed clamped-clamped beam, which is excited by an electro-dynamic shaker. The test-rig was designed to determine the linear and nonlinear parameters of the beam, but the shaker-structure interaction makes this a non-trivial task.

2 Experimental test

The test-rig is shown in figure 1 [4]. It consists of a pre-compressed clamped-clamped aluminium beam to give a structure with a hardening nonlinear cubic stiffness, a TIRA TV51120 shaker, two DYTRAN Accelerometers, and one DYTRAN force gauge.

The force applied to the beam was measured during the experiments so that the way in which the forces were distributed between the shaker and the beam as a function of frequency could be determined. Swept-sine tests were conducted, with a sweep rate of 0.05 Hz/s between 32 Hz and 45 Hz and with different levels of excitation (current source). The displacement envelopes for the centre and driven points of the beam (points B and A respectively in figure 1) for 140mV excitation were measured. The jump-down frequency at about 41 Hz can be observed. It can be seen that the response in the centre of the beam corresponds well to the Duffing system, but the response at the excitation point does not correspond as well.

Figure 1. Photograph showing the experimental setup for the beam-shaker system. A, Accelerometer 1; B, Accelerometer 2.

Conclusions

This paper has described an experimental study in which a nonlinear beam has been excited by an electrodynamic shaker. With such a configuration it is not possible to keep the force applied to the beam constant over a frequency range because of the changing impedance of the structure compared to the shaker. To interpret the experimental results correctly it is thus necessary to measure the force applied to the beam and to take the varying force amplitude into account in a nonlinear model of the beam.

Acknowledgements

The authors wish to acknowledge the financial support from the National Natural Science Foundation of China (No. 11672058).

References

Maximum Likelihood identification of Wiener-Hammerstein models in presence of process noise

Giuseppe Giordano  
Chalmers University of Technology  
giuseppe.giordano@chalmers.se  

Jonas Sjöberg  
Chalmers University of Technology  
sjoberg@chalmers.se  

1 Introduction

The Wiener-Hammerstein (W-H) model is a block-oriented model consisting of two linear blocks and a static non-linearity in the middle. Several identification approaches rely on the fact that the best linear approximation (BLA) of the system is a consistent estimate of the two linear parts, under the hypothesis of Gaussian excitation and in an output noise framework. Thus, the identification problem consists of splitting on the dynamics in the two linear parts and the estimation of the non-linearity. In this contribution, the presence of a disturbance entering before the nonlinearity (process noise) is considered. The W-H system with both presence of a disturbance entering before the nonlinearity and the estimation of the non-linearity. In this contribution, the presence of a disturbance entering before the nonlinearity (process noise) is considered. The W-H system with both measurement, $e(t)$, and process, $w(t)$, noise is defined by

\[ y(t) = G_2(q) f(x(t)) + e(t) \]  
\[ x(t) = x_0(t) + w(t) \]  
\[ x_0(t) = G_1(q) u(t) \]

where $G_1$ and $G_2$ are two LTI systems and $f$ describes a static non-linear function.

2 Standard prediction error method and maximum likelihood

The presence of process noise does not influence the consistency of the BLA of the system, but the estimation of the non-linearity, using a standard prediction error method (PEM), can lead to biased results, see [1]. Furthermore, a standard splitting algorithm based on the consistency of the estimation of the non-linearity, see the brute force algorithm in [2], cannot be used.

A new identification method, based on the Maximum Likelihood (ML) criterion, is derived. By extending the approach presented in [3] to the W-H case, the parameters of the two linear parts, $\alpha$, $\beta$, and the non-linearity, $\gamma$, can be obtained by minimizing the negative of the log likelihood function

\[ -\log(p_y(\alpha, \beta, \gamma; Z)) = \frac{N}{2} \log(\lambda_e, \lambda_w) - \sum_{t=1}^{N} \log e^{-\frac{1}{2} E(t; \alpha, \beta, \gamma)} dt(t) \]

where

\[ E(t; \alpha, \beta, \gamma) = \frac{1}{\lambda_e} (y(t) - G_2(q, \beta) f(x(t), \gamma))^2 \]

\[ + \frac{1}{\lambda_w} (x(t) - G_1(q, \alpha) u(t))^2 \]

where $\lambda_e$ and $\lambda_w$ are the measurement and process noise variances, which can be estimated too. Monte-Carlo integration methods and gradient-based optimization algorithms, using exact and approximated Hessian, are used to solve the optimization problem. The PEM has been compared to the ML on a low order W-H model, for the estimation of a third-degree polynomial function

\[ f(x(t), \gamma) = c_0 + c_1 x(t) + c_2 x^2(t) + c_3 x^3(t). \]

In Figure 1, the estimation of $c_1$, the parameter showing the highest bias when estimated with PEM, on 1000 data sets, is depicted.

The ML method provides an unbiased estimate of the non-linearity in presence of process noise. This, combined with a splitting algorithm based on the BLA, leads to the full identification of the W-H structure. Current work concerns the application of the ML method to the benchmark data provided by the Workshop on Nonlinear System Identification Benchmarks.

References

1 Wiener-Hammerstein system with process noise

Here, we will illustrate the capabilities and shortcomings of a kernel adaptive learning (KAL) approach applied to the one-step ahead prediction of the Wiener-Hammerstein (WH) system with process noise benchmark problem. The WH structure contains a static nonlinearity that is sandwiched in between two linear time invariant (LTI) blocks (Fig. 1). The major identification challenges associated with this benchmark problem are: (a) the process noise that is present in the system, (b) the static nonlinearity which is not directly accessible from neither the measured input nor output, (c) the output dynamics are difficult to invert due to the presence of a transmission zero.

2 Kernel adaptive learning

KAL algorithms fall into the class of online learning or system identification methods that build the solution to a regression or classification problem. Online learning methods update their solution iteratively. In the standard online learning framework, each iteration consists of several trials [1]. During the $n^{th}$ iteration: (a) The algorithm first receives an input datum, $x_n$, (b) Then, it calculates the estimated output $\hat{y}_n$ corresponding to this datum, (c) The true outcome $y_n$ is made available shortly thereafter, which enables the algorithm to calculate the loss $L(\cdot)$ incurred on the data pair $(x_n, y_n)$, (d) Finally, the solution is updated. A setup for online system identification with a kernel-based method is shown in Fig. 2. It represents an unknown nonlinear system (WH benchmark system with process noise in this case), whose input data $x_n$ and response $y_n$ (including additive noise $r_n$) can be measured at different time steps. Here, we use an adaptive kernel-based algorithm described in [2] to predict the system response.

3 Results

Fig. 3 shows the comparison of the measured (in red) and modelled response (in blue) on the data record created by concatenating the benchmark estimation and validation datasets respectively. The method is able to adapt to the changes in the data (dynamics) very efficiently, when used in the sequential prediction settings.

4 Acknowledgements

This work was supported in part by the IWT-SBO BATTLE 639, FWO-Vlaanderen, the Flemish Government (Methusalem), the Belgian Government through the Inter university Poles of Attraction (IAP VII) Program, and ERC advanced grant SNLSID under contract 320378.

References

A methodology to compare two estimation methods for Parallel Hammerstein Models

M. Rébillat\textsuperscript{1}, M. Schoukens\textsuperscript{2}

\textsuperscript{1}DYSO, PIMM, ENSAM, Paris, France, \textsuperscript{2}ELEC Department, Vrije Universiteit Brussel, Belgium

The problem addressed here is the estimation of nonlinear models of real life systems. A class of block-oriented models that is particularly considered here is the class of Parallel Hammerstein Models (PHM, see below).

One way to estimate PHM relies on the fact that the estimation problem is linear in the parameters and thus that least squares estimation algorithms can be used (DLS: direct least-squares method and RLS: regularized least-squares method \cite{Marconato2017}). Another means to estimate PHM consists in using exponential sine sweeps (ESS: parametric exponential sine sweep method and NP-ESS: non parametric exponential sine sweep method) and it is now well established that fully nonparametric versions of PHM can be very easily and rapidly estimated using it \cite{Rebillat2016}.

A methodology is proposed here to compare them with respect to their accuracy and their computational cost. Tests are performed on a simulated system for several values of methods respective parameters. The chosen system is the Bouc-Wen system that belongs to the benchmarks proposed at the ”Workshop on Nonlinear System Identification Benchmarks” held in Brussels in 2016 \cite{Noel2016}. The sampling frequency is chosen as $f_s = 2500$ Hz for this system. It is known that the system to be identified does not correspond exactly to a PHM.

In order to compare both methods, computational time as well as two performance indexes have been considered. $P_{I1}$ is the relative difference between the reconstructed signal and the output signal for an exponential sine sweep input. $P_{I2}$ is the relative difference between the reconstructed signal and the output signal for a noise input. On the result figure, one point presents the result obtained by one method, for one run and for a given combination of free parameter without any additional post-processing. The indications [\% > $\alpha$] in the legend indicate the percentage of point that lie out of the figure. A large gap in computational time exists between the two classes of methods. The different methods provide ESS reconstruction results having very similar precisions but at a much lower cost for ESS based methods. There is an impact of the input signal on the performances of the LS-based methods highlighting the fact that the estimated model is valid only for a certain “class”.

References


Interpolated Linear Modeling of the F16 Benchmark

Maarten Schoukens
Department of ELEC, Vrije Universiteit Brussel, B1050 Brussels, Belgium
maarten.schoukens@vub.ac.be

1 Introduction

The F16 Benchmark system [1] contains multiple resonances. These resonances shift in function of the input signal amplitude. Estimating a nonlinear model that captures this shifting resonance behavior can result in very complex models with a high number of parameters. This abstract makes use of interpolated linear time invariant (LTI) models [2] to capture the shifting resonance behavior observed in the F16 Benchmark.

2 Interpolated Linear Modeling

In a first step a set of root LTI models is estimated on a number of system setpoints. In a second step these LTI models can be combined using an interpolating scheme to obtain the modeled system behavior on a previously unobserved system setpoint. Of course, one can think of many different LTI system representations and interpolation schemes [2]. Here, a continuous time rational transfer function model is used, where the numerator and denominator coefficients are interpolated linearly to obtain the system model at a unobserved setpoint.

3 Results

The $F16Data_{FullMSine}_{Levelx}$ datasets are used for the estimation of the root LTI models. A set of 4 12-th order continuous-time rational transfer functions are obtained (see Figure 1). The models are validated on the validation datasets $F16Data_{FullMSine}_{Levelx,Validation}$. The rms value of the input is used as the interpolating weight.

The resulting interpolated models are validated on the frequency response function obtained from the validation datasets. Figure 2 illustrate the good performance of the interpolated model on the $F16Data_{FullMSine}_{Level4,Validation}$ dataset. The interpolated model residuals are compared with the residuals obtained by fitting a LTI model on the validation dataset directly. It can be observed that the residuals of the interpolated and the estimated validation LTI model are almost coinciding in for most frequencies, a significant difference can be observed in the first two resonances.

4 Conclusion

Interpolated linear models are successfully capturing the moving resonance behavior observed in the F16 benchmark. However it cannot describe the actual underlying nonlinear behavior causing this shift in resonance.

Acknowledgment

This work was supported in part by the ERC advanced grant SNL-SID 320378.

References

Transient elimination and memory saving possibilities for multidimensional nonparametric regularization illustrated on the cascaded water tanks benchmark problem

Péter Zoltán Csurcsia\textsuperscript{1,2}, Georgios Birpoutsoukis\textsuperscript{1}, Johan Schoukens\textsuperscript{1}
1) Vrije Universiteit Brussel, Department of Fundamental Electricity and Instrumentation
Pleinlaan 2, B-1050 Elsene, Belgium
2) Siemens Industry Software NV
Interleuvenlaan 68, B-3001 Leuven, Belgium
Peter.Zoltan.Csurcsia@vub.ac.be, Georgios.Birpoutsoukis@vub.ac.be, Johan.Schoukens@vub.ac.be

1 Introduction

In this work an efficient nonparametric time domain (nonlinear) system identification method is presented which provides 1) a nonlinear modeling tool, 2) a transient estimation technique, and 3) memory reduction possibilities for the regularization technique. The proposed work is illustrated on the cascaded tanks measurement benchmark \cite{1}.

2 The nonparametric identification method

2.1 The model structure

Assume that the dynamics of an underlying (nonlinear) system can be described by the following finite discrete time Volterra series of degree $M$ \cite{2} extended with the transient term $y_{tr}[n]$:

$$y[n] = h_0 + \sum_{m=1}^{M} \sum_{\tau_1=0}^{n_1-1} \cdots \sum_{\tau_m=0}^{n_m-1} h_m[\tau_1, \ldots, \tau_m] \prod_{j=0}^{m} u[n - \tau_j] + y_{tr}[n] + e[n]$$

where $u[n]$ denotes the input, $y[n]$ represents the measured output signal, $e[n]$ is zero mean i.i.d. white noise with finite variance, $h_m[\tau_1, \ldots, \tau_m]$ is the Volterra kernel of order $m$, and $\tau_i, i = 1, \ldots, m$ denotes the lag variables.

2.2 The cost function

Equation (1) can be rewritten into a vectorial form as

$$Y = K\theta + E,$$

where $\theta$ contains the Volterra and transient coefficients, $K$ is the observation matrix, $Y$ contains the measured output, and $E$ contains the measurement noise. The kernel coefficients are obtained by minimizing the following (regularized least square) cost function \cite{3}:

$$\hat{\theta}_{reg} = \arg \min \{ ||Y - \theta K||^2 + \theta^T D \theta \}$$

where the block diagonal matrix $D$ contains $(M + 1)$ submatrices penalizing the coefficients of the Volterra kernels and the transient impulse response function. In this work, prior information about the smoothness and the exponential decay is used during the identification step by proper construction of matrix $D$ \cite{2}.

2.3 Results and conclusions

1) In case of long measurements and/or large number of estimated parameters the memory needs can be excessive. To avoid the exaggerated memory usage, in this work a practical gradient-based estimation method is presented, leading to the same numerical results as the proposed Volterra estimation method.

2) The transient effects in the simulated output are removed by a special regularization method based on the novel ideas of transient removal for Linear Time-Varying (LTV) systems \cite{3}.

Combining the proposed methodologies, it is clear that the obtained models capture the system dynamics when tested on a validation dataset, and their performance is comparable with the white-box (physical) models.

3 Acknowledgments

This work was supported by FWO-Vlaanderen, the Methusalem project, IAP VII) Program, by the ERC advanced grant SNLSID, under contract 320378, and by the VLAIO Innovation Mandate project number HBC.2016.0235.

References

\cite{1} M. Schoukens, P. Mattson, T. Wigren, J.P. Nol, Cascaded tanks benchmark combining soft and hard nonlinearities, Workshop on Nonlinear System Identification Benchmarks, 2016, Brussels

\cite{2} G. Birpoutsoukis and J. Schoukens, Regularized nonparametric Volterra kernel estimation, IEEE I2MTC, 11-14 May 2015

SYSTEM IDENTIFICATION OF DYNAMIC FORCE TRANSUDCERS

Saheb R. Hassan

Physikalisch-Technische Bundesanstalt, Bundesallee 100, 38116 Braunschweig, Germany, saheb.r.hasan.ext@ptb.de

1. INTRODUCTION

In recent years, many national metrology institutes have developed dynamic calibration of force transducers [1]–[4]. The main goal of the calibration of force transducers is to maintain their parameters, such as stiffness and damping coefficient, in addition to their sensitivity. In this contribution, we introduce a mathematical model of the calibration of dynamic force transducers and their system identification using Levenberg-Marquadt algorithm. The force transducer mounted to an electrodynamic shaker, which applies a periodic chirp excitation. A load mass is attached on the top of the force transducers in order to increase the applied inertial force amplitudes. The acceleration measured on two points, on the top of the shaker armature and on the top surface of the load mass.

2. SYSTEM MODEL

The calibration setup can modelled as a series of mechanical oscillators as illustrated in Figure 1, each oscillator represents the different elastic connections that existing in the system. In addition, the most significant oscillator is the one that represents the force transducer itself with a stiffness, \( k_f \) and a damping, \( b_f \). Therefore, we can simplify the system to either one or two Degree-of-freedom system with the transfer functions \( G(s) \) shown in equations (1), (2) respectively. The transfer functions have poles because we consider a displacement input to the system \( y_s(t) \).

\[
\frac{Y_f(s)}{Y_s(s)} = \frac{b_f s + k_f}{m_f s^2 + b_f s + k_f} \tag{1}
\]

\[
\frac{Y_f(s)}{Y_s(s)} = \frac{b_f s + k_f}{(b_f s + k_f) + m_b s + k_a} = \frac{b_f s + k_f}{m_b s^2 + (b_f + k_b) s + k_a} \tag{2}
\]

3. MODEL AND EXPERIMENTAL DATA

Figure 2 shows preliminary results of the system identification of force transducers using two different load masses using the proposed 1DOF model in equation (1).

Figure 1: Experimental data and model fits with two different load masses 3 and 7 kg. (black curves represent measured data and coloured curves represent fit model)

Figure 2: Different models of force transducer with additional mass and applied to shaker with different degrees of freedom (DOF). 5 DOF on the right, three DOF in the middle, one DOF in the lift
4. CHALLENGES TO SYSTEM IDENTIFICATION OF DYNAMIC FORCE TRANSDUCERS

The current approach of modelling dynamic force transducers as linear time variant system with neglecting nonlinearities is facing several challenges such as:

1- Nonlinear hysteresis behaviour of force transducer.

![Figure 3: Calibration data show the hysteresis of force transducer (continuous line for loading and dash lines for unloading)](image)

2- Nonlinear frequency-dependent damping

3- Nonlinear displacement-dependent stiffness

![Figure 4: force-deflection curve shows nonlinear stiffness behaviour. The deflection is measured using Laser interferometer. The load applied statically using a primary method.](image)

4- The absence of the shake armature dynamic parameters in addition to the difficulty of measuring the input acceleration/displacement.

5. REFERENCES


Decoupled polynomial nonlinear state space models of a Bouc-Wen hysteretic system

Alireza Fakhrizadeh Esfahani\textsuperscript{1} \hspace{1cm} Philippe Dreesen\textsuperscript{1} \hspace{1cm} Jean-Philippe Noël\textsuperscript{1,2} \hspace{1cm} Koen Tiels\textsuperscript{1} \\
Johan Schoukens\textsuperscript{1}

\textsuperscript{1} Vrije Universiteit Brussel, ELEC Department, firstname.middlename.lastname@vub.be \\
\textsuperscript{2} University of Liège, Space Structures and Systems Laboratory, Aerospace and Mechanical Engineering Department, jp.noel@ulg.be

1 Introduction

The benchmark data is generated from a Bouc-Wen model. A polynomial nonlinear state space (PNLSS) model is estimated. Then, by using canonical polyadic decomposition (CPD), the number of parameters in the PNLSS model is reduced significantly. Many models of different orders are presented, so the user has freedom to make a compromise between flexibility and parsimony. The main goal of this study is looking on the behavior of all possible decoupled models for the Bouc-Wen hysteresis.

2 PNLSS Model

The PNLSS model is a state space model that includes all possible monomials in the states and input up to a certain power (p):
\[
\begin{align*}
    x(t+1) &= Ax(t) + bu(t) + E \zeta(x(t),u(t)), \\
    y(t) &= c^T x(t) + du(t) + f^T \eta(x(t),u(t))
\end{align*}
\]

For example, for \( p = 2 \) we have
\[
\zeta(x,u) = \eta(x,u) = [x_1^2 \ x_1 x_2 \ x_1 u \ x_2^2 \ x_2 u \ u^2].
\]

The PNLSS model is initialized with a linear state space model that is obtained by using the linear state space identification approach in [1] on the best linear approximation [2] of the system. All the parameters of the PNLSS model are optimized with a Levenberg-Marquardt algorithm [3].

3 Decoupled Models

To reduce number of parameters, the nonlinear part of the PNLSS model is decomposed into a linear transformation \( V \), followed by a set of parallel univariate polynomials \( g_1, \ldots, g_r \), and another linear transformation \( W \), resulting in the decoupled model:
\[
\begin{align*}
    x(t+1) &= Ax(t) + bu(t) + W x g^T [x(t) \ u(t)], \\
    y(t) &= c^T x(t) + du(t) + W y g^T [x(t) \ u(t)]
\end{align*}
\]

The decoupling is calculated in the following way [4]
\begin{enumerate}
    \item Evaluate the Jacobian of \( E \zeta^T f^T \eta \) in some sampling points \((x,u)\) drawn from a Gaussian distribution.
    \item Stack the Jacobians in a tensor \( J \)
    \item Compute the CPD [5] of \( J \) which gives the \( V^T, W \), and \( H \) (containing the derivatives of \( g \) matrices).
    \item Find the univariate polynomials \( (g_1, \ldots, g_r) \) from \( H \).
\end{enumerate}

4 Results

For 3 branches \((r = 3)\) and degree of 10 for the univariate polynomials (the best found model) gives 2.91 dB lower rms error than the full PNLSS model (PNLSS: -94.56dB, Decoupled model: -97.47dB) on the test data. This is done for the different possible numbers of branches \((r)\) with different orders of nonlinearity in branches.

5 Acknowledgement

This work was supported in part by the Fund for Scientific Research (FWO-Vlaanderen) and by the ERC advanced grant SNLSID, under contract 320378.

References

The Decoupled Polynomial NARX Model: Parameter Reduction and Structural Insights for the Bouc-Wen Benchmark

David Westwick  
University of Calgary  
dwestwic@ucalgary.ca

Gabriel Hollander  
Vrije Universiteit Brussel  
gabriel.hollander@vub.ac.be

Johan Schoukens  
Vrije Universiteit Brussel  
Johan.Schoukens@vub.ac.be

1 Introduction

The polynomial NARX model is defined by:

\[ y(t) = \sum_{k=1}^{K} \gamma_k \phi_k(u, y) + e(t), \quad (1) \]

where the \( \phi_k(u, y) \) are monomial functions of the current and past inputs, stored in the vector \( u \) and the past outputs, in \( y \), the \( \gamma_k \) are coefficients and \( e(t) \) is an IID noise sequence. As evident in (1), it is linear in the variables. However, the number of variables increases combinatorially with the polynomial degree, and the number of past inputs and outputs included in the model. Typically, an orthogonal forward regression algorithm is used to prune the model [1].

Following [2], the multivariate polynomial will be replaced by a small number of univariate polynomials, each of which transforms a linear combination of the variables:

\[ y(t) = \sum_{r=1}^{R} g_r(v_r^T [u \ y]), \quad (2) \]

where the \( g_r(\cdot) \) are univariate polynomial functions, and \( R \) is the number of branches in the model. Note that the vectors \( v_r \) can be interpreted as pairs of FIR filters, where half filters the past inputs, and the other half filters the past outputs.

2 Results

We generated identification data from the default set-up of the Bouc-Wen Benchmark [3], and identified the 7 models listed in Table 1. All models used 8 past outputs and 8 past inputs. The sigmoidnet NARX model included a direct feedthrough term. All polynomial models were of degree 3. The models were then tested in both simulation and one-step-ahead prediction on the two test signals provided with the benchmark. Simulation errors are reported in Table 1.

Figure 1 shows the 4 univariate nonlinearities from the 4-branch model, with the linear terms suppressed. The curves have been normalized, and then slightly offset from each other to facilitate comparison. All of the polynomial terms appear to include a deadzone like behaviour.

3 Conclusion

The decoupled polynomial NARX model provides better simulation accuracy while using fewer parameters than any of the other NARX structures tested. The structure of the model also provides some insight into the mechanisms present in the system.

References


<table>
<thead>
<tr>
<th>Model Description</th>
<th># of Param</th>
<th>MultiSine (dB)</th>
<th>SweptSine (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>16</td>
<td>-76.9</td>
<td>-74.1</td>
</tr>
<tr>
<td>Full Poly. NARX</td>
<td>969</td>
<td>∞</td>
<td>∞</td>
</tr>
<tr>
<td>Pruned NARX</td>
<td>90</td>
<td>-84.1</td>
<td>-90.8</td>
</tr>
<tr>
<td>Sigmoidnet NARX</td>
<td>803</td>
<td>-83.0</td>
<td>-88.7</td>
</tr>
<tr>
<td>Decoupled, 2 branches</td>
<td>38</td>
<td>-81.4</td>
<td>-82.5</td>
</tr>
<tr>
<td>Decoupled, 3 branches</td>
<td>57</td>
<td>-82.8</td>
<td>-86.1</td>
</tr>
<tr>
<td>Decoupled, 4 branches</td>
<td>76</td>
<td>-84.3</td>
<td>-93.8</td>
</tr>
</tbody>
</table>

Table 1: Number of Model Parameters, and RMS Simulation Error for all identified models.

Figure 1: Univariate nonlinearities from the 4 branch model. Note that the linear terms have been suppressed.