Study of the effective number of parameters in nonlinear identification benchmarks

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Wiener-Hammerstein benchmark

![Graph showing the Wiener-Hammerstein benchmark with RMSE vs. number of parameters for different models. The models include BLA, poly3-WH, poly10-WH, poly12-WH, poly17-WH, NN-NLSS, FS-LSSVM, LSSVM, SA-PNLSS, PWL8-WH, PWL30-WH, PWL24-WH, and PNLSS.]
$n_{eff}$ vs. $n_\theta$

$n_\theta$ Number of parameters
- SVMs?
- Regularization?

$n_{eff}$ Effective number of parameters
- Property of the identified model
- Degrees of freedom in the model parametrization
Outline

- $n_{\text{eff}}$ vs. $n_{\theta}$
  - Motivation example: FIR case
  - Linear / Nonlinear in the parameters
  - Comparison on WH benchmark
Motivation: FIR example

\[ \hat{y} = \sum_{k=0}^{d} \hat{g}_k u(t - k) \]
Motivation: FIR example

\[
\hat{y} = \sum_{k=0}^{d} \hat{g}_k u(t - k)
\]

System response

Least squares solution

\[
\hat{g} = (K^T K)^{-1} K^T y
\]
Motivation: FIR example

\[ \hat{y} = \sum_{k=0}^{d} \hat{g}_k u(t - k) \]
Motivation: FIR example

\[ \hat{y} = \sum_{k=0}^{d} \hat{g}_k u(t - k) \]

\[ \hat{g} = (K^T K)^{-1} K^T y = V \Sigma^{-1} U^T y = V \theta \]

SVD

\[ K = U \Sigma V^T \]
Motivation: FIR example

\[ \hat{g} = V\theta \]

\[ \hat{g} = \tilde{V}\tilde{\theta} \]

- \( n_\theta \times 1 \)
- \( n_\theta \times 1 \)
- \( n_\theta \times 1 \)
- \( n_{\text{eff}} \times 1 \)

System response

Truncated solution

Least squares solution

Magnitude

\[ k \]
LINEAR REGRESSION

$$\hat{y} = K\theta$$
$$\hat{\theta} = (K^TK)^{-1}K^Ty = V\Sigma^{-1}U^Ty$$

SVD
$$K = U\Sigma V^T$$

Regressor matrix and $n_{\text{eff}}$

Rank $K$ ↓ $n_{\text{eff}}$ ↓
Jacobian matrix and $n_{\text{eff}}$

\[
\Delta \theta = (J^T J)^{-1} J^T e = V \Sigma^{-1} U^T e
\]

\[
\hat{\theta}_{i+1} = \hat{\theta}_i + \Delta \theta
\]

\[J = U \Sigma V^T\]

$\text{Rank } J \downarrow \rightarrow n_{\text{eff}} \downarrow$
WH results: comparison

\[ n_{\theta} = 134 \]

\[ \text{RMSE} = 5.6 \text{ mV} \]
WH results: singular values of J

![Graph showing singular values and a threshold](image)
WH results: $n_{\text{eff}}$

REGULARIZATION (ridge regression)

\[ n_{\text{eff}} = \sum_{i=1}^{n_{\theta}} \frac{\sigma_i^2}{\sigma_i^2 + \lambda} \]

\[ \sigma^2 \]

\[ \lambda = 1 \]

\[ n_{\text{eff}} = 33 \]

Magnitude [dB]

Singular value number
WH results: comparison

![Graph showing RMSE vs. Effective number of parameters for various models including BLA, poly3-WH, poly10-WH, poly12-WH, poly17-WH, regularized NN-NLSS, PWL8-WH, PWL30-WH, PWL24-WH, and NN-NLSS. The numbers represent RMSE values for each model.]
Conclusion

- Effective number of parameters
  - Measure of model complexity for a given dataset
- More correct comparison of nonlinear models
  - WH benchmark
- Rank reduced estimation based on truncated SVD
  - NL in the parameters: still open problem
Thank you for your attention!

Any questions?
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Silverbox results: comparison

\[ n_\theta = 23 \]

\[ \text{RMSE} = 0.34 \text{ mV} \]
Silverbox results: comparison

![Graph showing RMSE vs Effective number of parameters with different marker labels like BLA, NLFB, regularized NN-NLSS, poly-LFR, and NN-NLSS.](Image)