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Study of the effective number of parameters in nonlinear identification benchmarks

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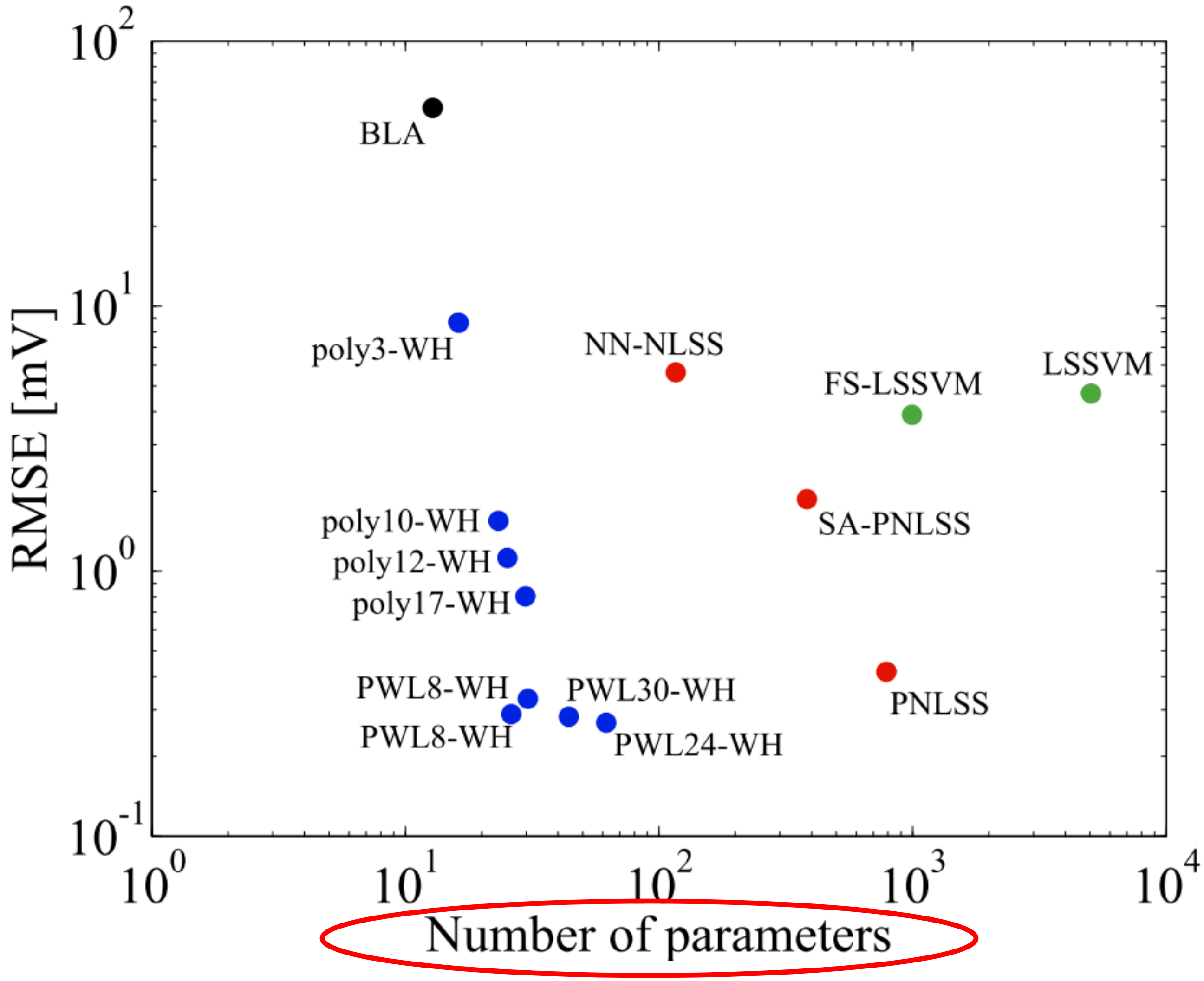
Wiener-Hammerstein benchmark



n_{eff} vs. n_θ

n_θ **Number of parameters**

- SVMs?
- Regularization?

n_{eff} **Effective number of parameters**

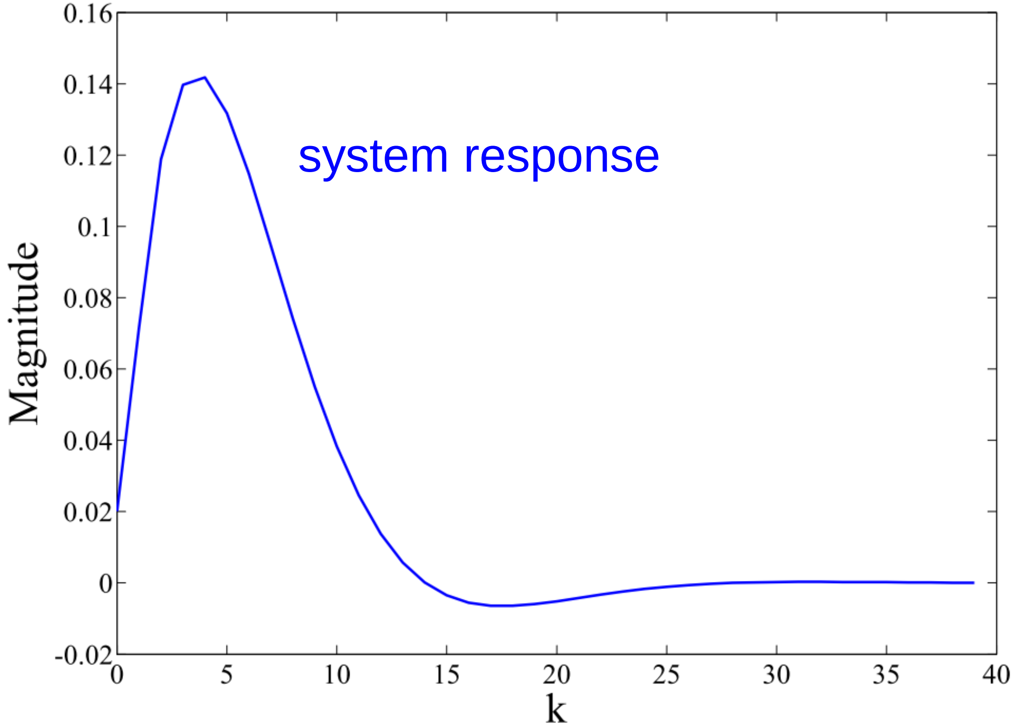
- Property of the identified model
- Degrees of freedom in the model parametrization

Outline

- n_{eff} **vs.** n_θ
  - Motivation example: FIR case
  - Linear / Nonlinear in the parameters
  - Comparison on WH benchmark

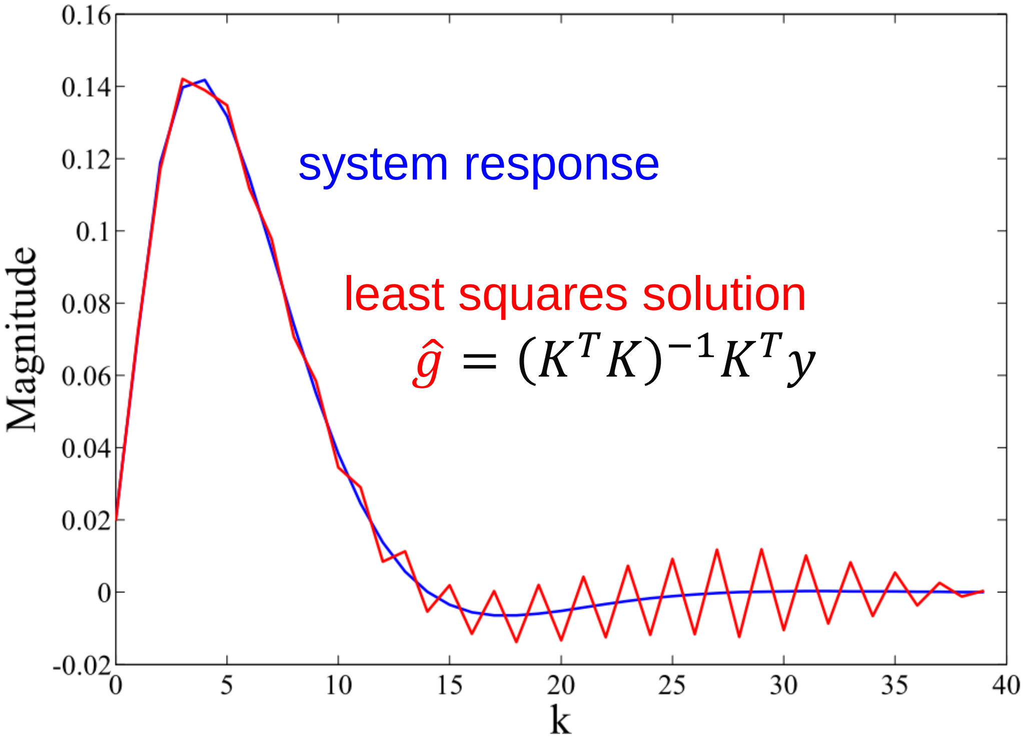
Motivation: FIR example

$$\hat{y} = \sum_{k=0}^{d} \hat{g}_k u(t-k)$$



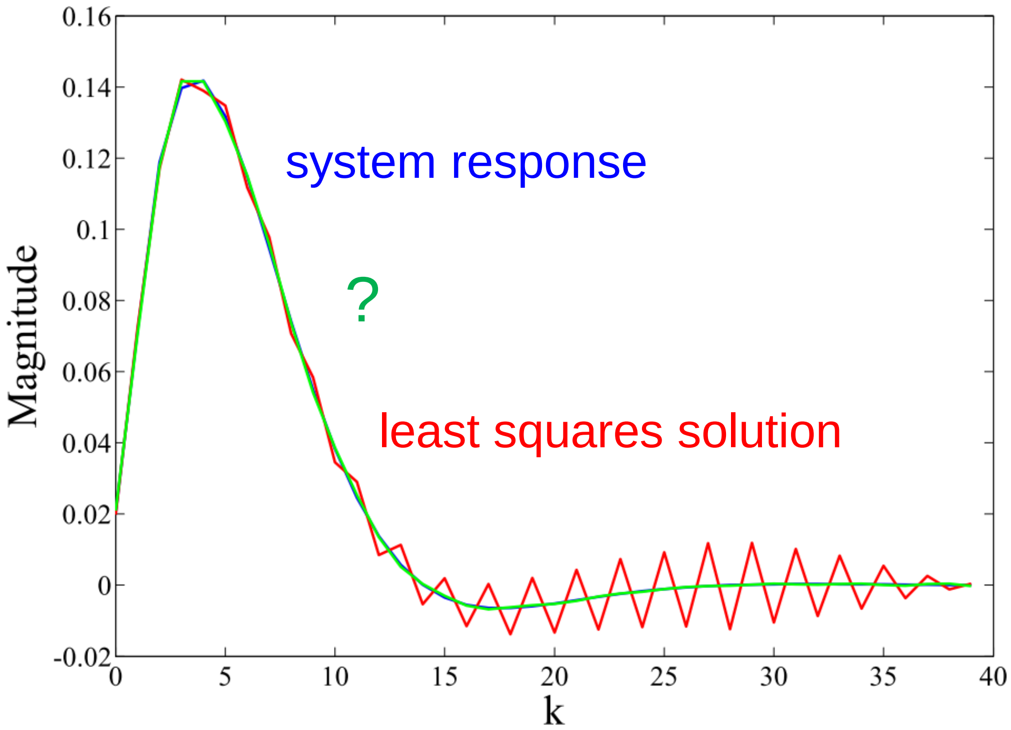
Motivation: FIR example

$$\hat{y} = \sum_{k=0}^{d} \hat{g}_k u(t-k)$$



Motivation: FIR example

$$\hat{y} = \sum_{k=0}^{d} \hat{g}_k u(t-k)$$



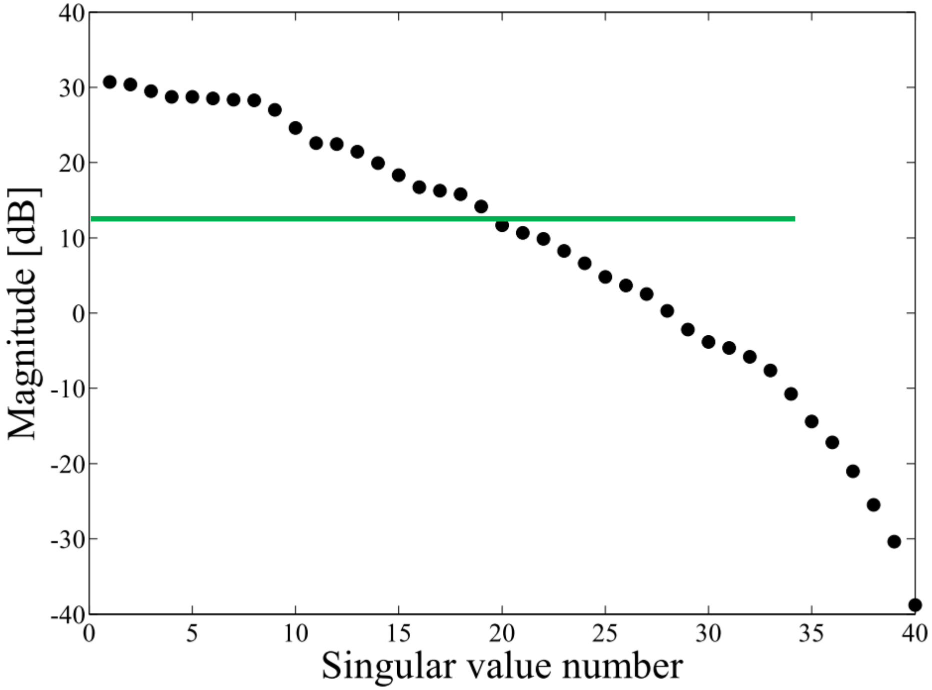
Motivation: FIR example

$$\hat{y} = \sum_{k=0}^{d} \hat{g}_k u(t-k)$$

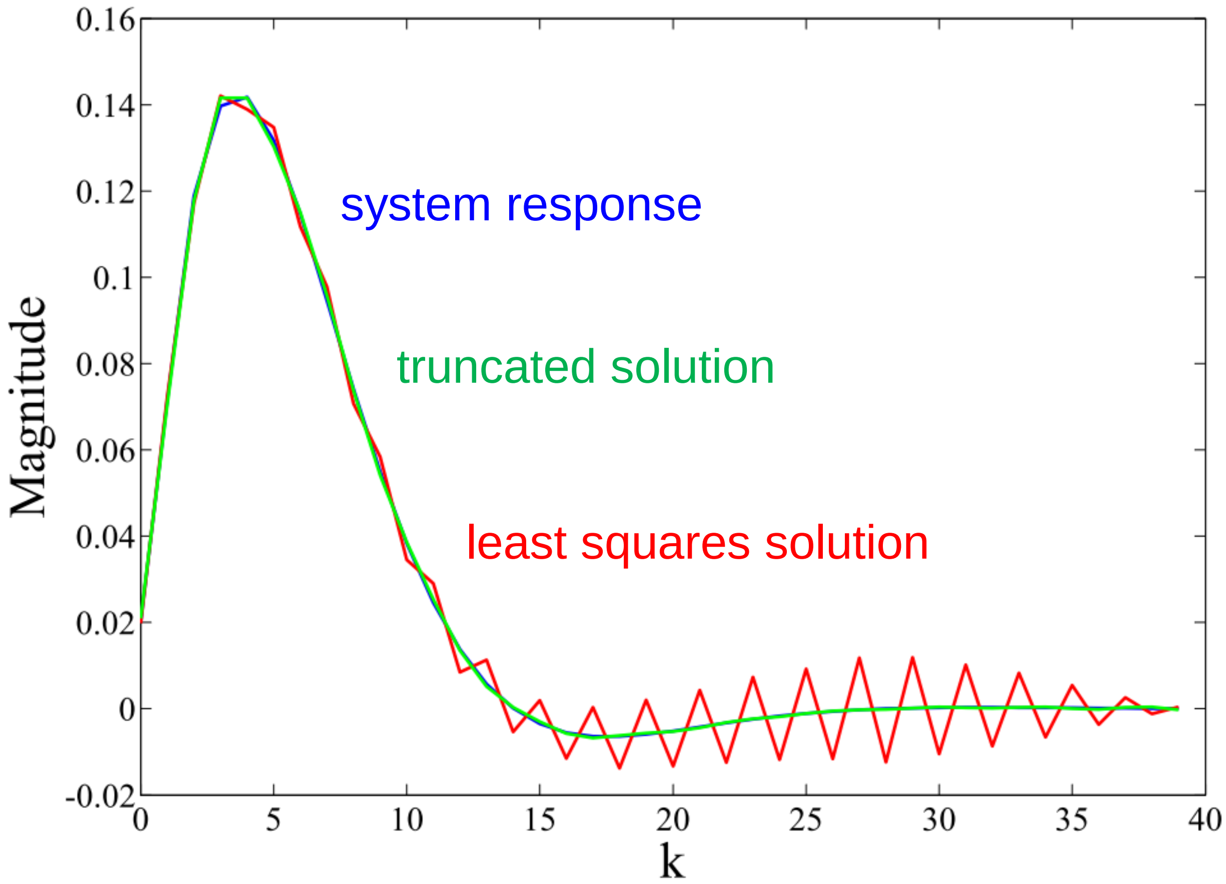
$$\hat{g} = (K^T K)^{-1} K^T y = V\Sigma^{-1} U^T y$$
$$= V\theta$$

SVD

$$K = U\Sigma V^T$$



Motivation: FIR example



$$\boxed{\hat{g} = V\theta}$$

$n_\theta \times 1$ $n_\theta \times 1$

$$\boxed{\hat{g} = \tilde{V}\tilde{\theta}}$$

$n_\theta \times 1$ $n_{\mathrm{eff}} \times 1$

Regressor matrix and n_{eff}

LINEAR REGRESSION

$$\hat{y} = K\theta$$

$$\hat{\theta} = \boxed{(K^T K)^{-1} K^T} y = V\Sigma^{-1} U^T y$$

SVD

$K = U\Sigma V^T$

Rank K ↓ ➡ n_{eff} ↓

Jacobian matrix and n_{eff}

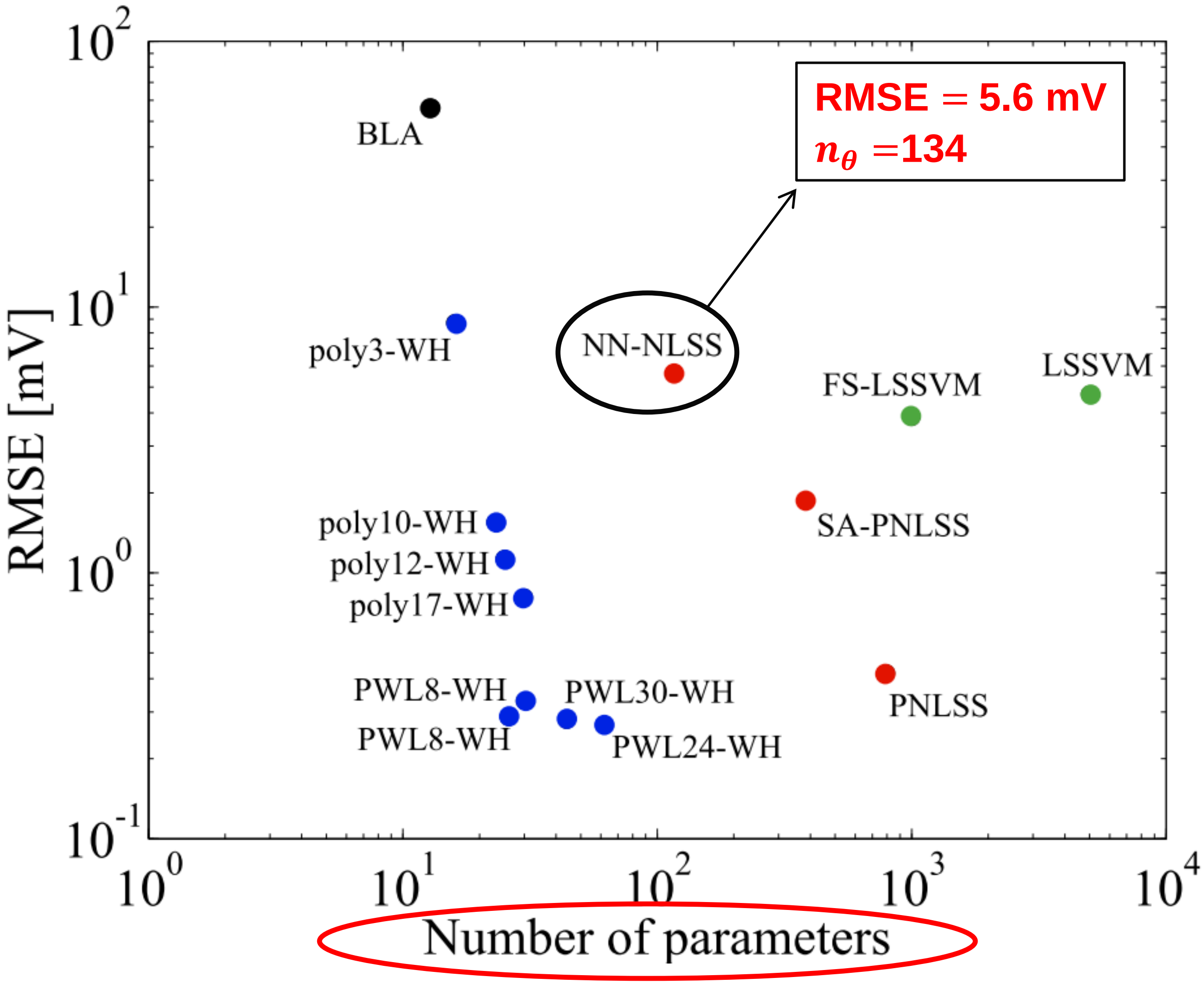
NONLINEAR IN THE PARAMETERS

$$\Delta\theta = \boxed{(J^T J)^{-1} J^T} e = V\Sigma^{-1}U^T e$$
$$\hat{\theta}_{i+1} = \hat{\theta}_i + \Delta\theta$$

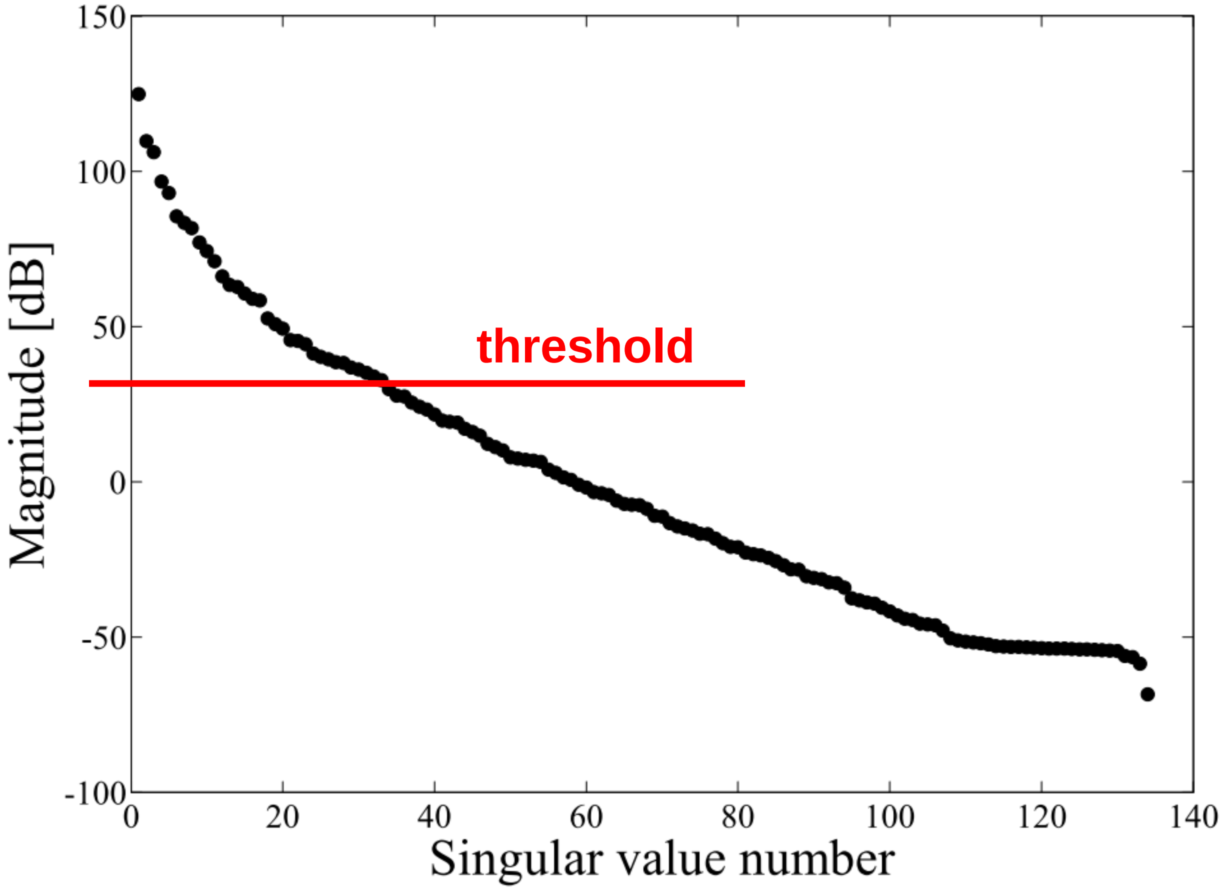
SVD
$J = U\Sigma V^T$

Rank *J* $\downarrow$ ➡ $n_{\mathrm{eff}} \downarrow$

WH results: comparison



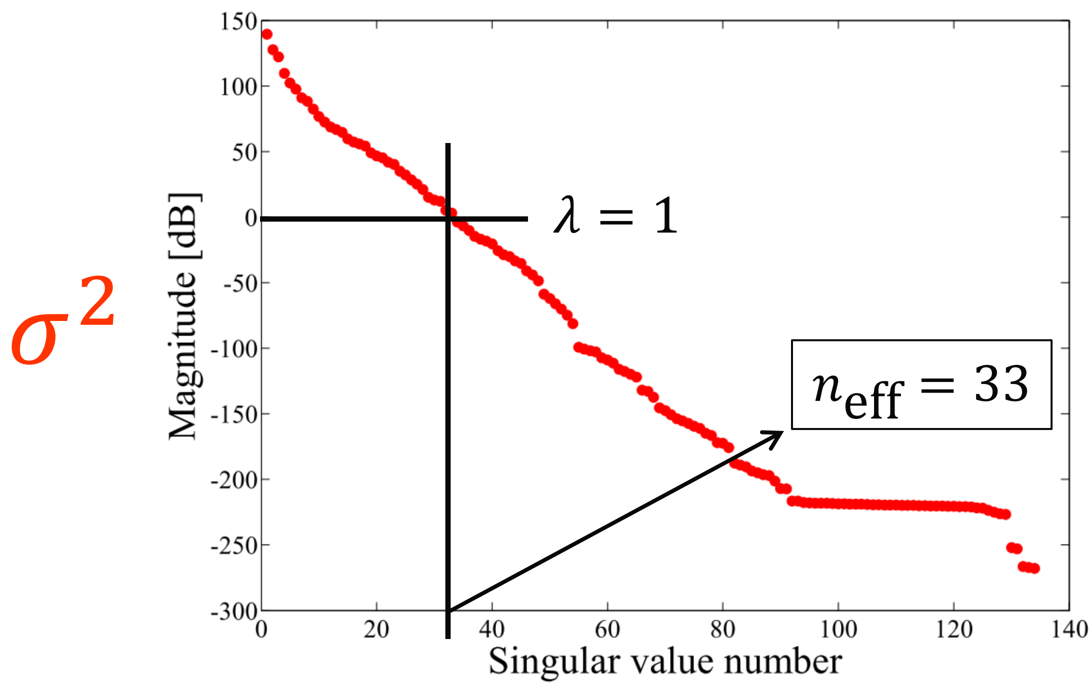
WH results: singular values of J



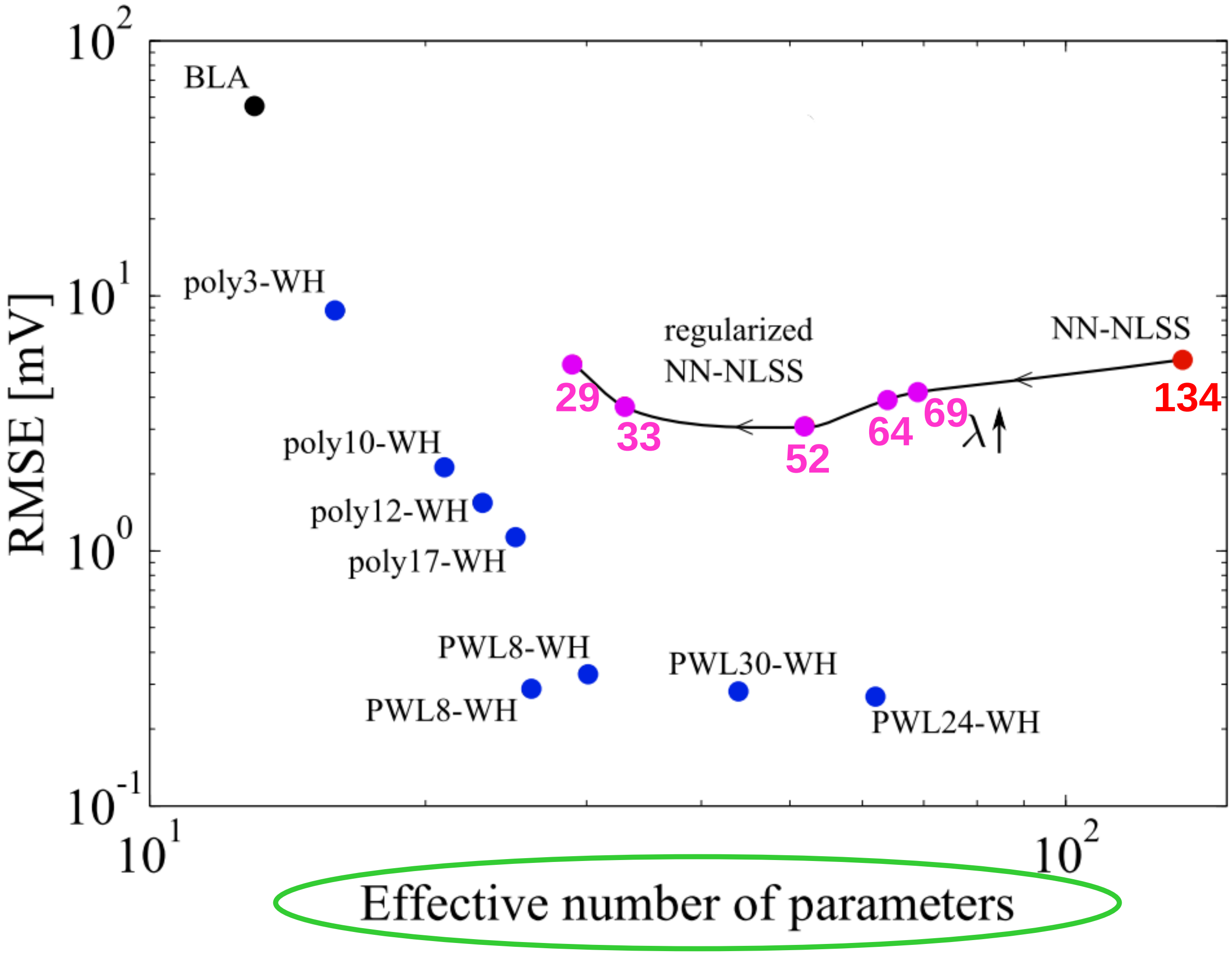
WH results: n_{eff}

REGULARIZATION (ridge regression)

$$n_{\text{eff}} = \sum_{i=1}^{n_\theta} \frac{{\sigma_i}^2}{{\sigma_i}^2 + \lambda}$$



WH results: comparison



Conclusion

- Effective number of parameters
  - Measure of model complexity for a given dataset
- More correct comparison of nonlinear models
  - WH benchmark
- Rank reduced estimation based on truncated SVD
  - NL in the parameters: still open problem

Thank you for your attention!

Any questions?

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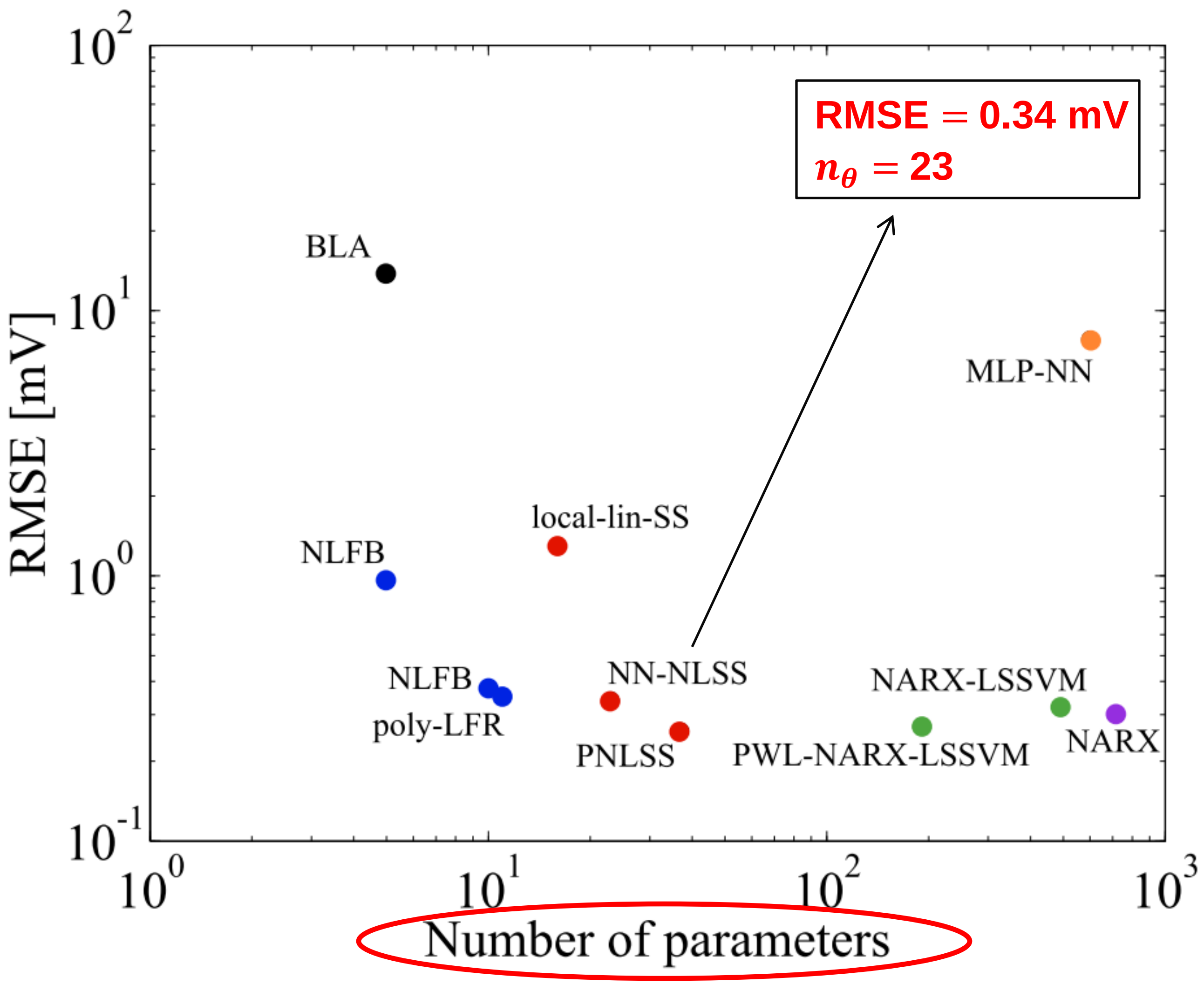
Silverbox results: comparison



Silverbox results: comparison