Identification of Wiener-Hammerstein systems with process noise using an Errors-in-Variables framework

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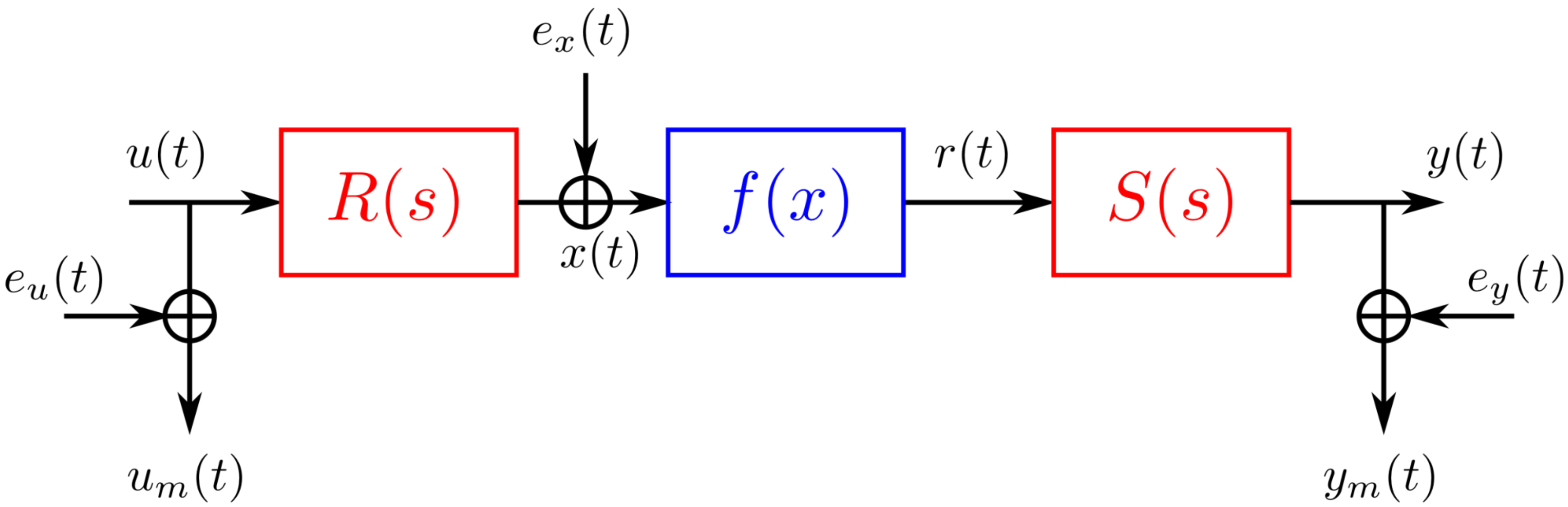
Benchmarks



Overview

Best Linear Approximation

Wiener-Hammerstein – Output-Error

Influence of the process noise?

Wiener-Hammerstein – EIV

Results

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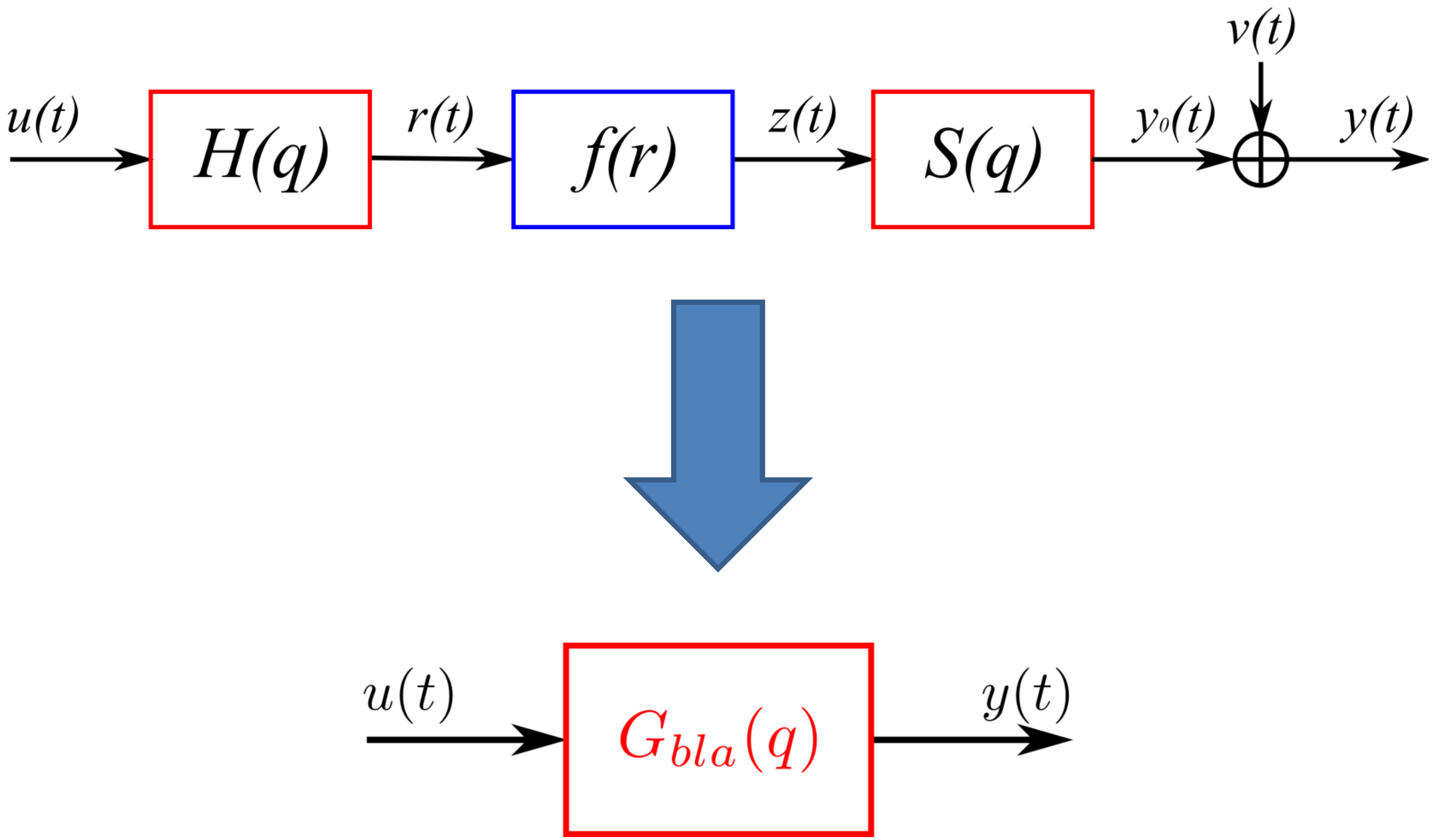


Bussgang's Theorem

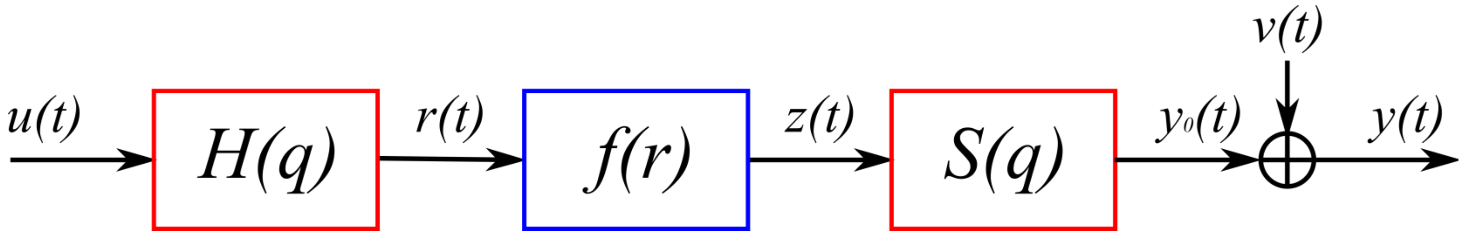
Stationary Gaussian input

→ Static nonlinearity ≈ static gain

$$f(u) = \gamma u$$

Structure detection

Wiener-Hammerstein



$$G_{bla}(q) = \gamma H(q)S(q)$$

➔ Only gain factor

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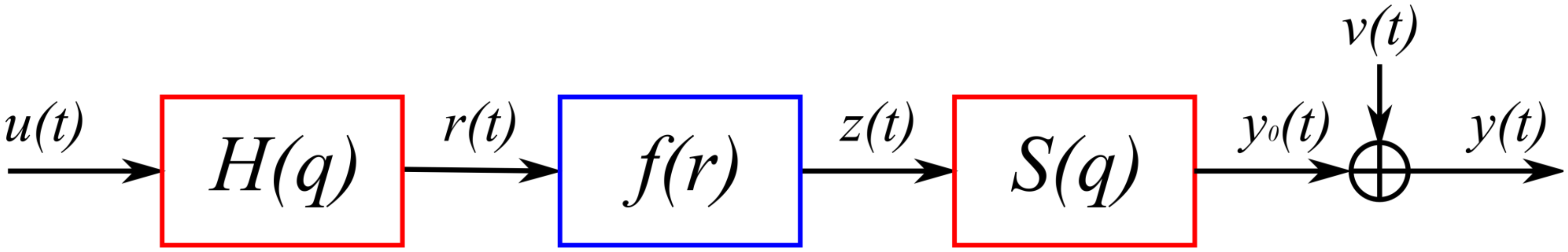
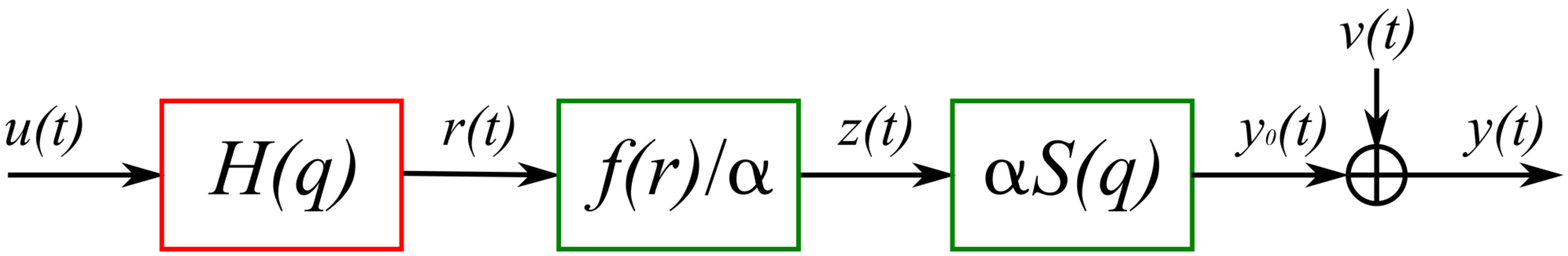
Wiener-Hammerstein: OE

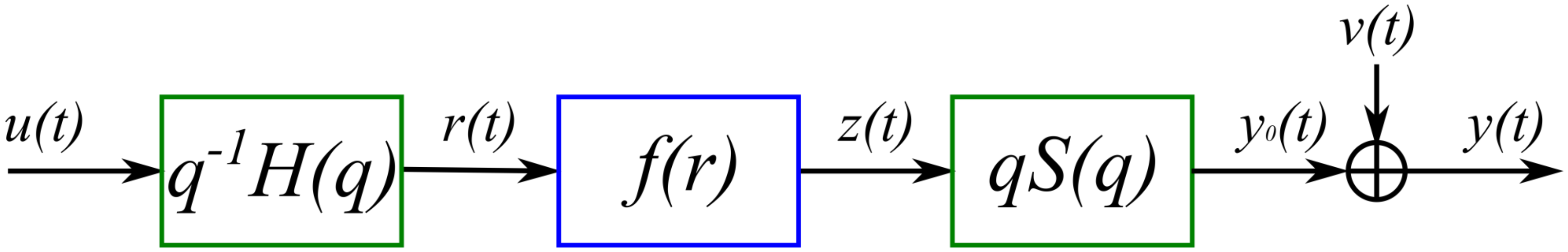


Identifiability



Gain exchange

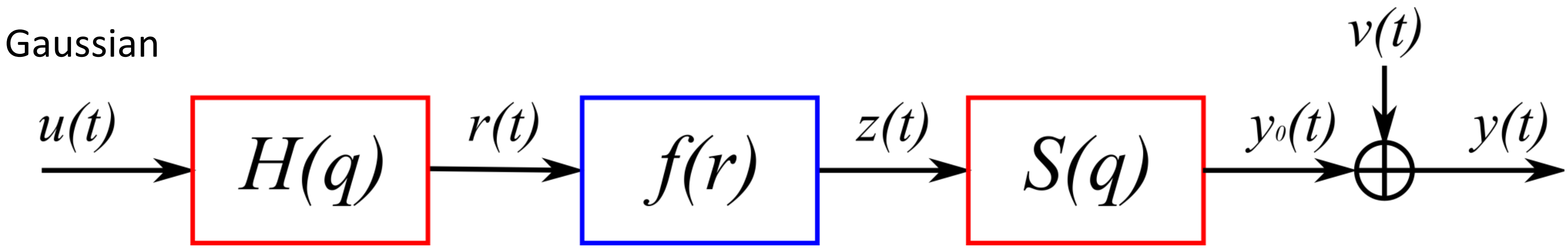
Identifiability



Gain exchange

Delay exchange

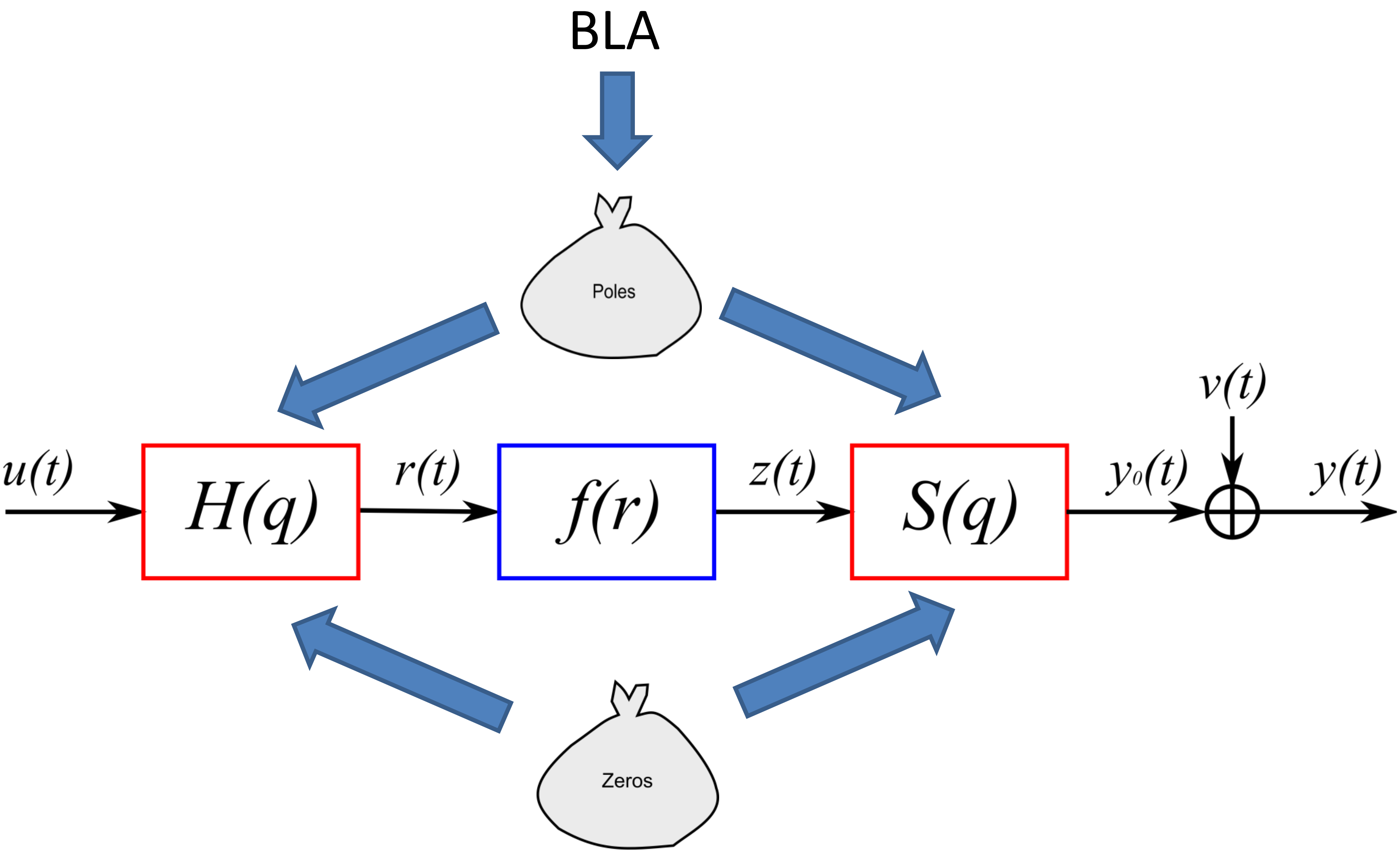
Best Linear Approximation



$$G_{bla}(q) = \gamma H(q)S(q)$$

➔ poles, zeros BLA = poles, zeros system

Partition the Dynamics



Nonlinear optimization

Initial parameter values

→ Optimization of all parameters together

→ Levenberg-Marquardt algorithm

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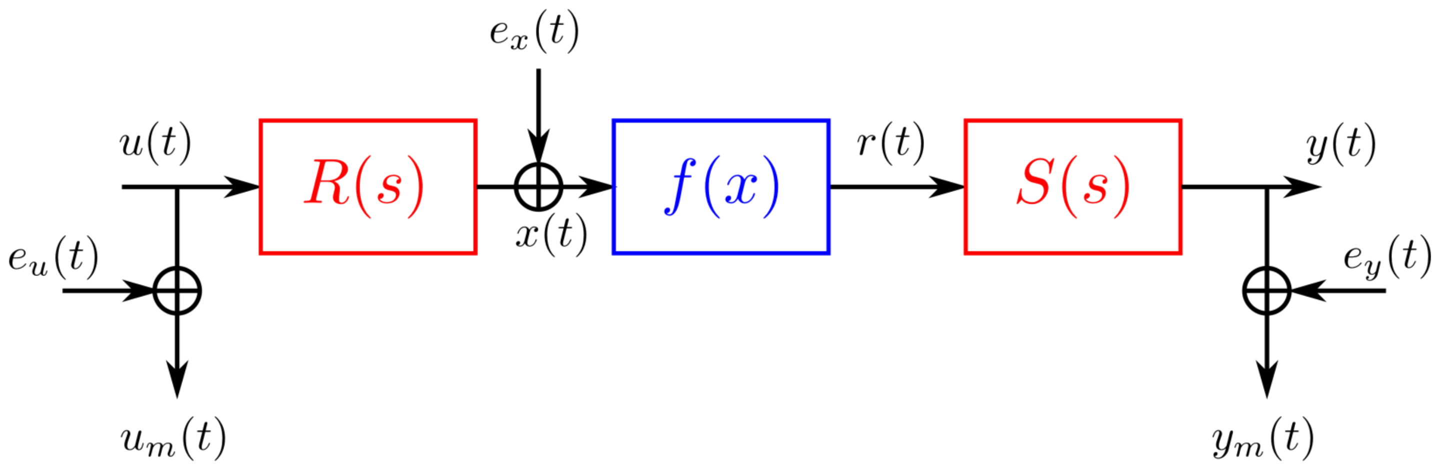
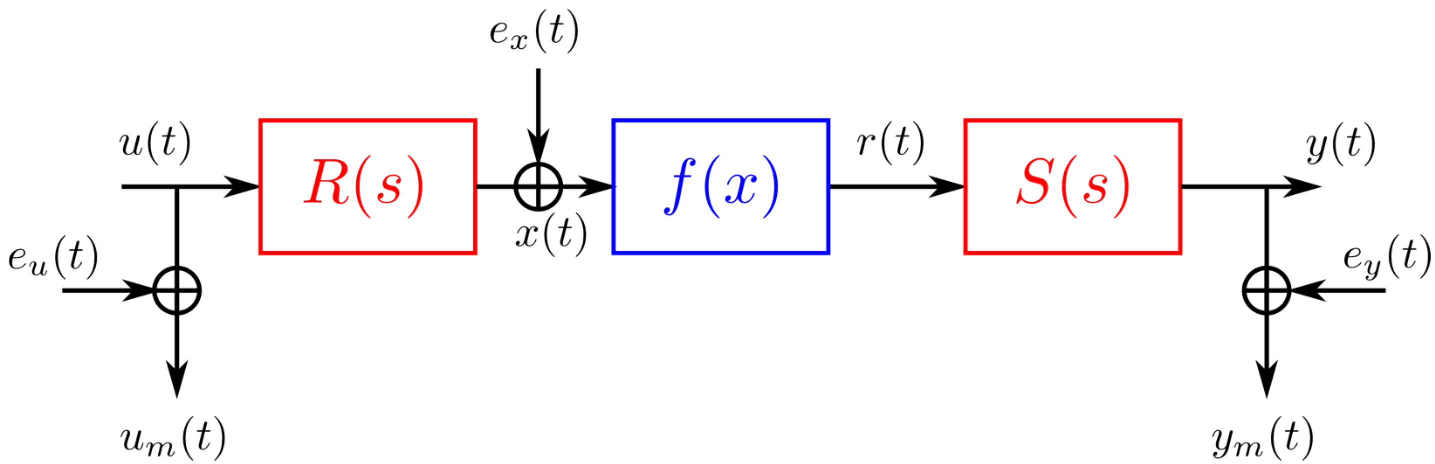
Influence of the Process Noise



$e_x(t)$: Gaussian ⇒ $x(t)$: Gaussian ⇒ Bussgang's Theorem

$$G_{bla}(s) = \gamma S(s) R(s)$$

Influence of the Process Noise



$e_x(t)$: Gaussian ⇒ $x(t)$: Gaussian ⇒ Bussgang's Theorem

$$G_{bla}(s) = \gamma S(s)R(s)$$

$x(t)$ depends on $e_x(t)$ ⇒ γ depends on $e_x(t)$

Example: 3rd Degree NL

$$y = (x_0 + e_x)^3$$
$$= {x_0}^3 + 3e_x {x_0}^2 + 3{e_x}^2 x_0 + {e_x}^3$$

Assumptions:

- Gaussian
- Zero-mean
- Independent

$$\gamma = \frac{E\{y x_0\}}{E\{x_0 x_0\}}$$
$$= \frac{E\{{x_0}^4 + 3e_x {x_0}^3 + 3{e_x}^2 {x_0}^2 + {e_x}^3 x_0\}}{E\{{x_0}^2\}}$$
$$= \frac{E\{{x_0}^4 + 3{e_x}^2 {x_0}^2\}}{E\{{x_0}^2\}}$$
$$= \frac{3{\sigma_x}^4 + 3{\sigma_x}^2 {\sigma_e}^2}{{\sigma_x}^2}$$
$$= 3{\sigma_x}^2 + 3{\sigma_e}^2$$

Bias due to odd nonlinear terms

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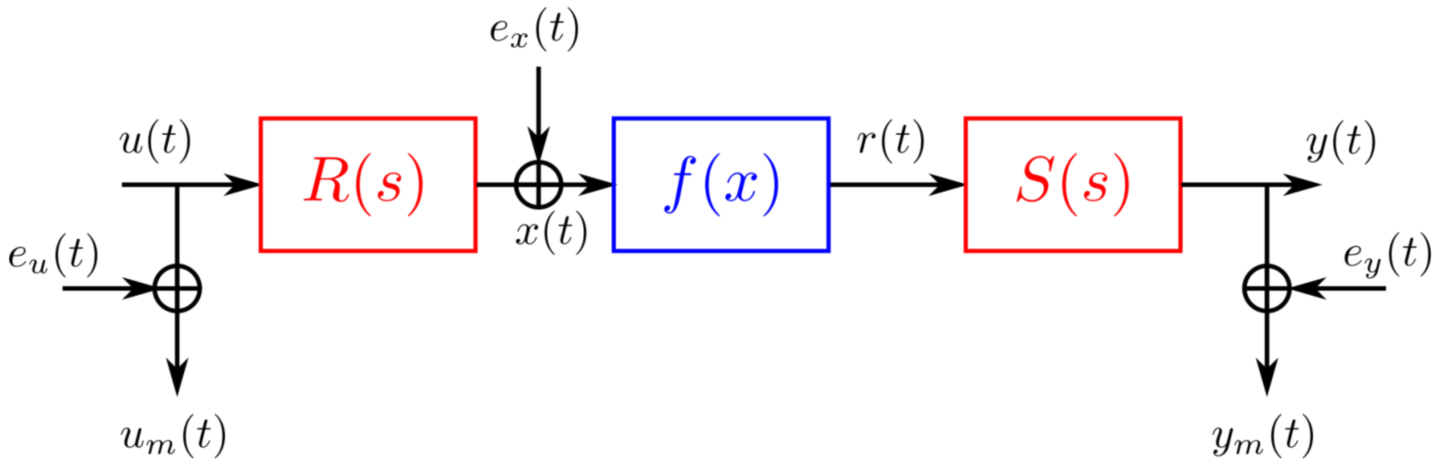
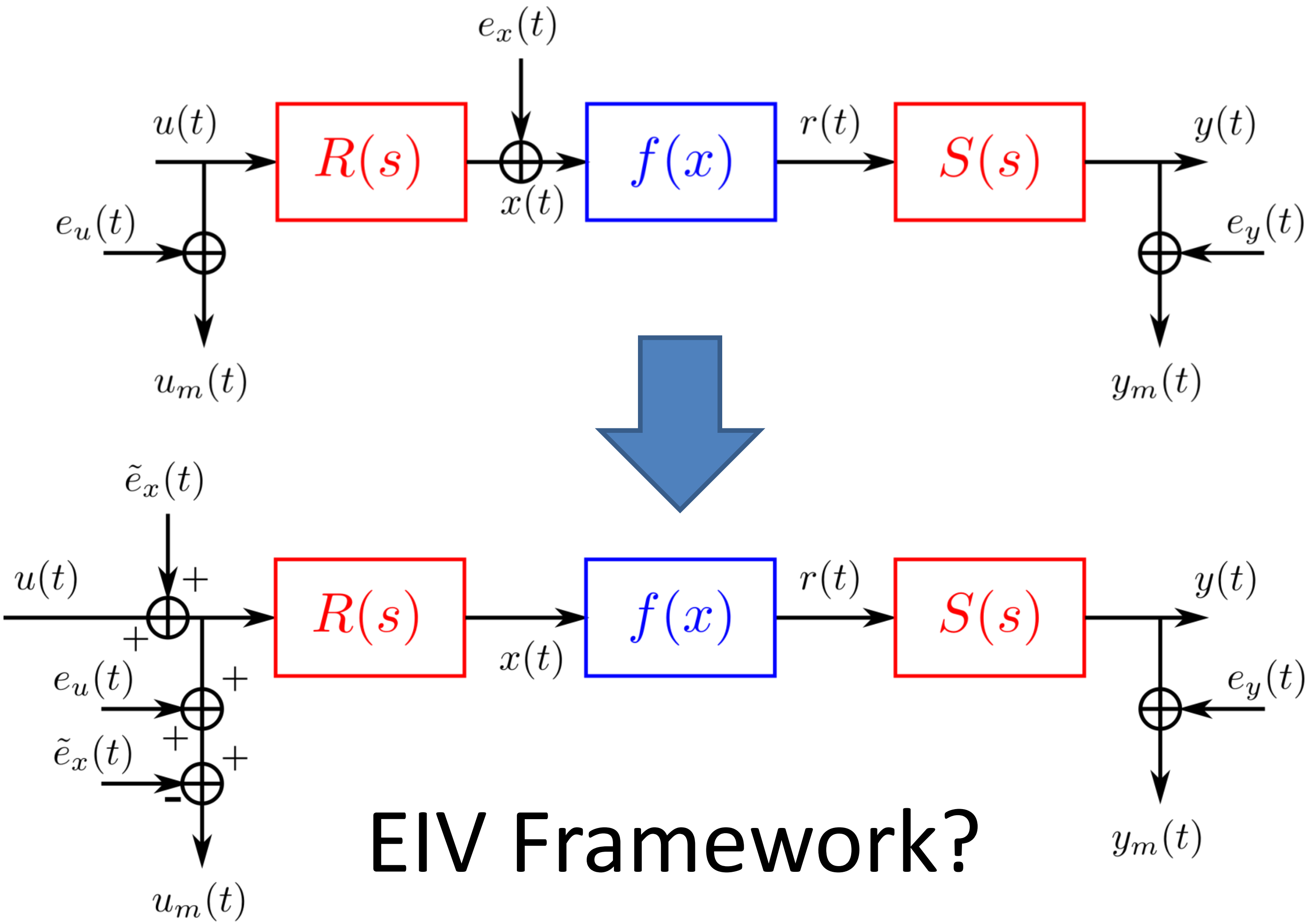
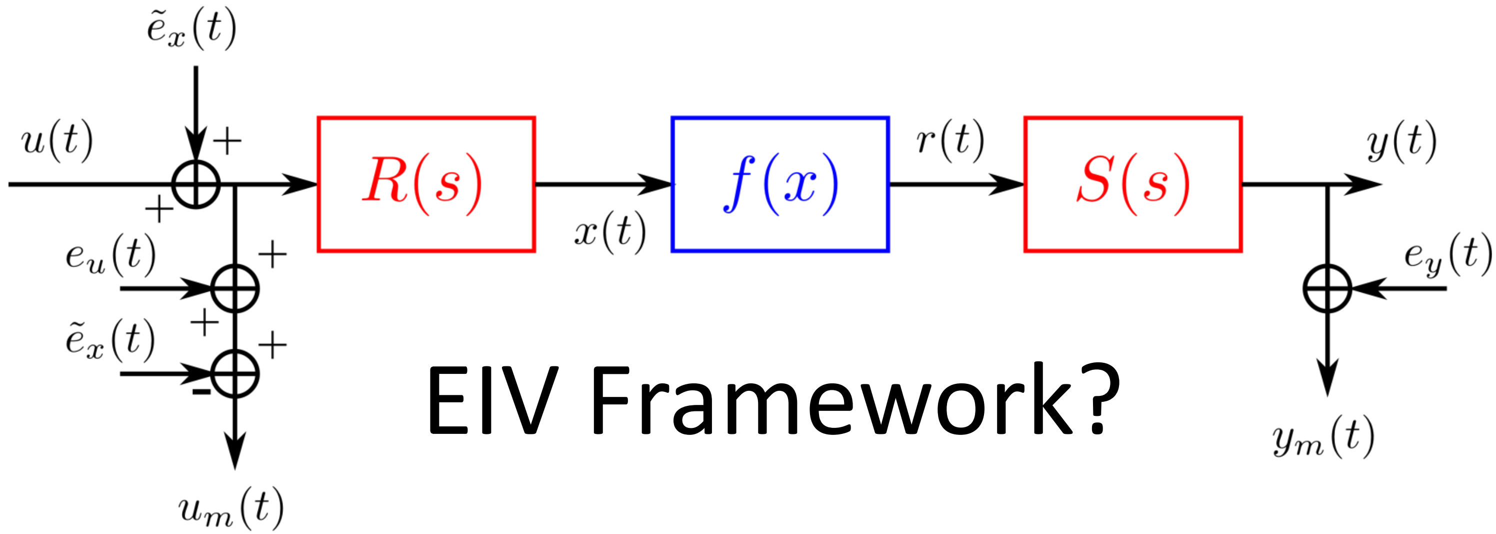


Wiener-Hammerstein: EIV



EIV Framework?

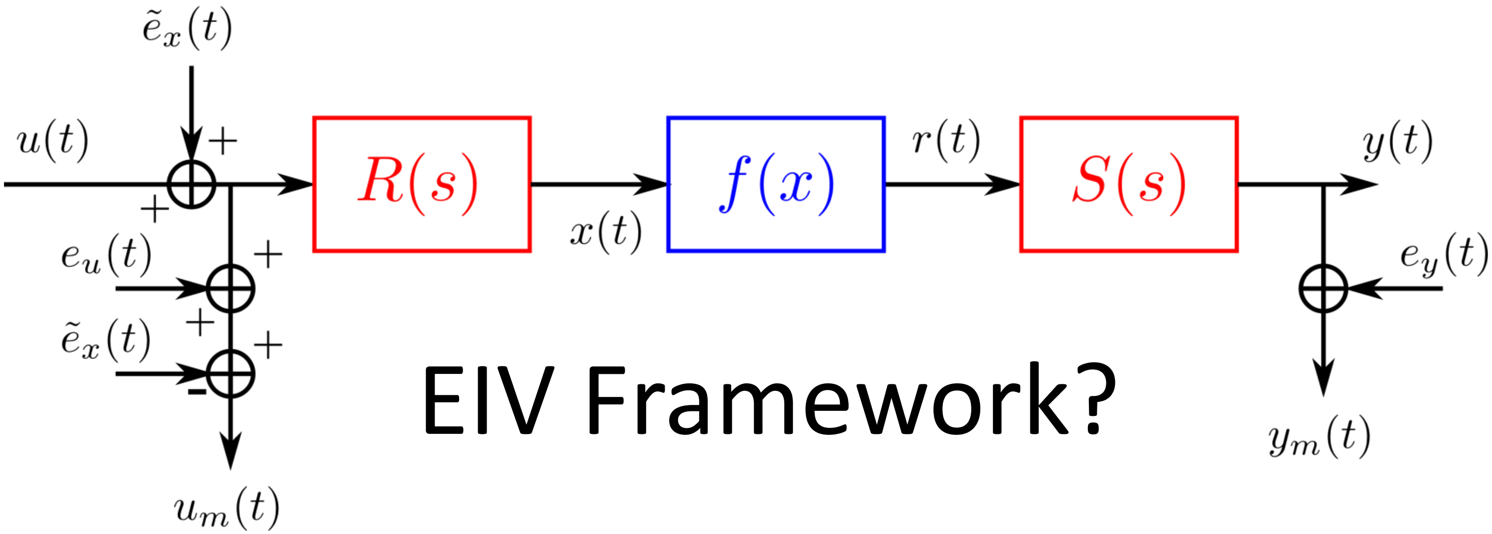
Wiener-Hammerstein: EIV



EIV Framework?

Output depends on input noise → Bias!

Wiener-Hammerstein: EIV



EIV Framework?

Let us try it anyway:

$$V_N(\theta) = \frac{1}{N}\sum_{t=1}^{N}\left(\frac{(u(t)-\hat{u}(t))^2}{\sigma_{e_u}^2} + \frac{(y(t)-\hat{y}(t))^2}{\sigma_{e_y}^2}\right) + \lambda\left(\hat{\sigma}_{e_u} - \sigma_{e_u}\right)^2,$$

Wiener-Hammerstein: EIV

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Direct optimization of input on selected freq.

Penalty term introduces prior knowledge

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Results

Estimation Data

Random Phase Multisine Input:

- frequencies: 0-15 kHz
- RMS: 0.7581
- 4096 Samples
- 1 Period
- 10 Realizations
- fs: 78.125 kHz

Results

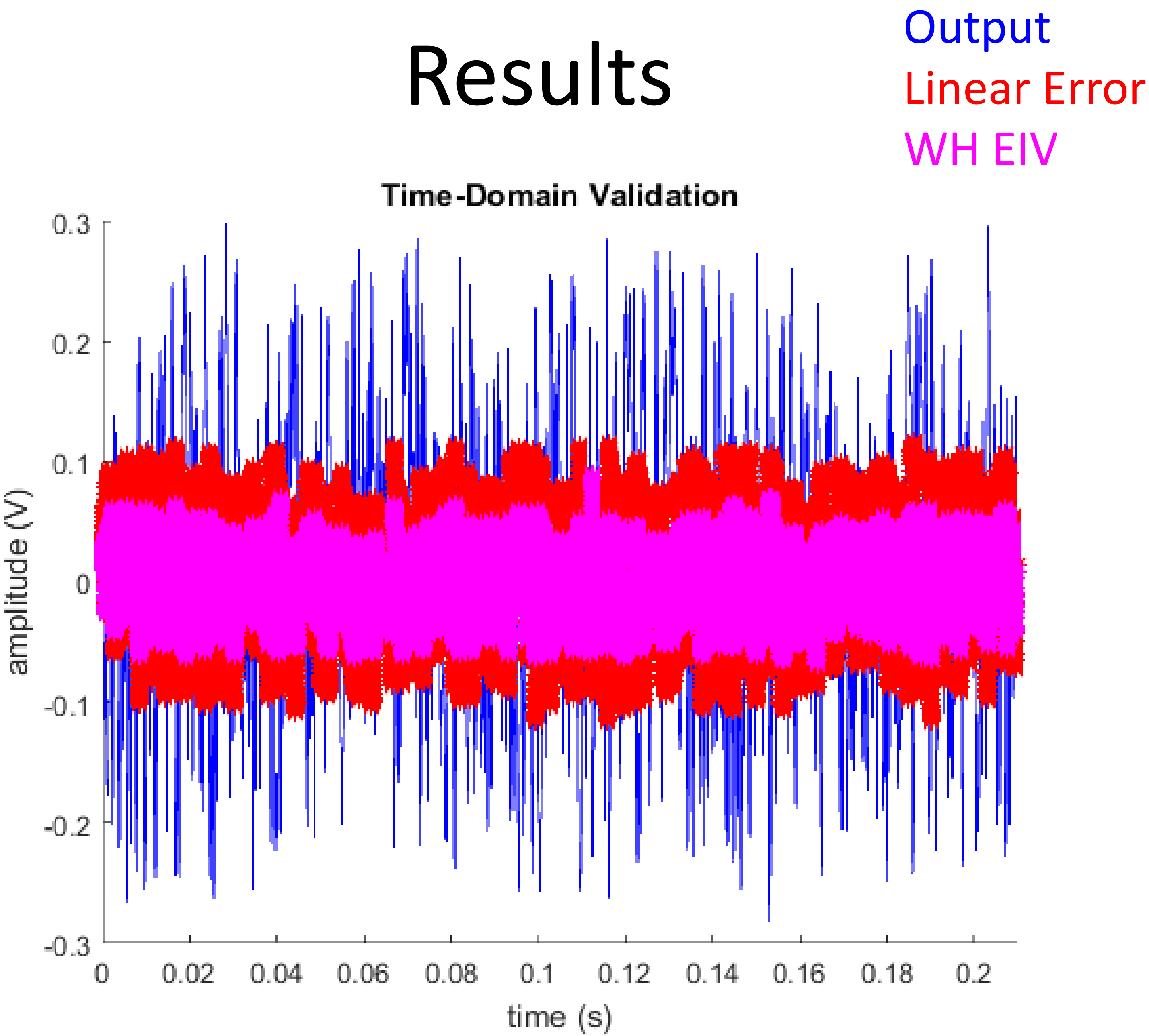
BLA: order 6/6

Wiener-Hammerstein:

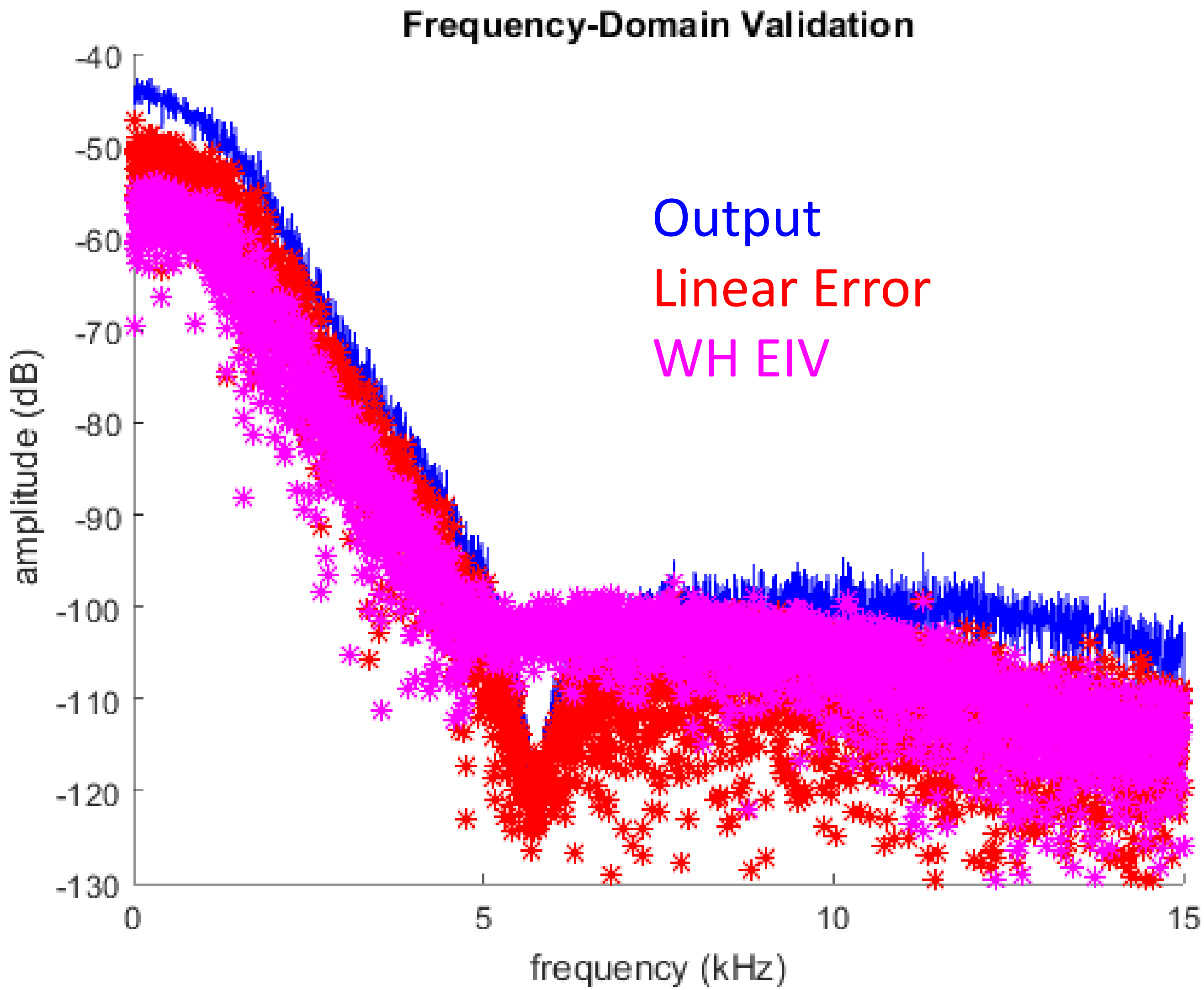
Neural Network

3 tansig activation functions

Results


Output
Linear Error
WH EIV
Time-Domain Validation
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
amplitude (V)
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
time (s)

Results



Frequency-Domain Validation
-40
-50
-60
-70
-80
-90
-100
-110
-120
-130
amplitude (dB)
0
5
10
15
frequency (kHz)
Output
Linear Error
WH EIV

Results

Simulation – Validation/Test Results

Multisine

	LTI	WH OE	WH EIV
Realization 1	0.055875	0.031387	0.022458
Realization 2	0.055875	0.031387	0.023080
Realization 3	0.055875	0.031387	0.040724
Realization 1-3	0.055875	0.031113	0.025004

Results

Simulation – Validation/Test Results

Sinesweep

	LTI	WH OE	WH EIV
Realization 1	0.019492	0.01485	0.039964
Realization 2	0.019492	0.01485	0.02139
Realization 3	0.019492	0.01485	0.022443
Realization 1-3	0.019492	0.011967	0.01913

Conclusions

Process noise introduces a bias through odd NL terms

Identification with process noise is not just an EIV problem

EIV methods can result in better estimates