Modeling Nonlinear Systems Using a Volterra Feedback Model

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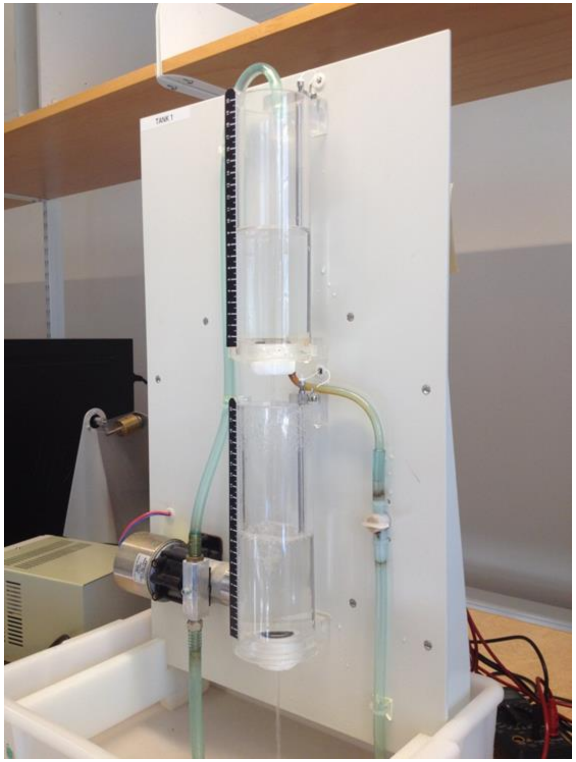
Benchmarks

Cascaded Tanks

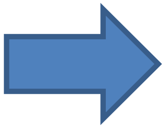


Bouc-Wen

$$m_L \ddot{y}(t) + r(y, \dot{y}) + z(y, \dot{y}) = u(t)$$

$$r(y, \dot{y}) = k_L y + c_L \dot{y}$$

$$\dot{z}(y, \dot{y}) = \alpha \dot{y} - \beta(\gamma |\dot{y}| z + \delta \dot{y} |z|)$$

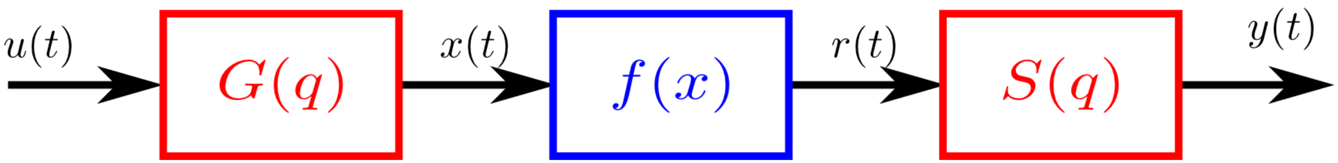


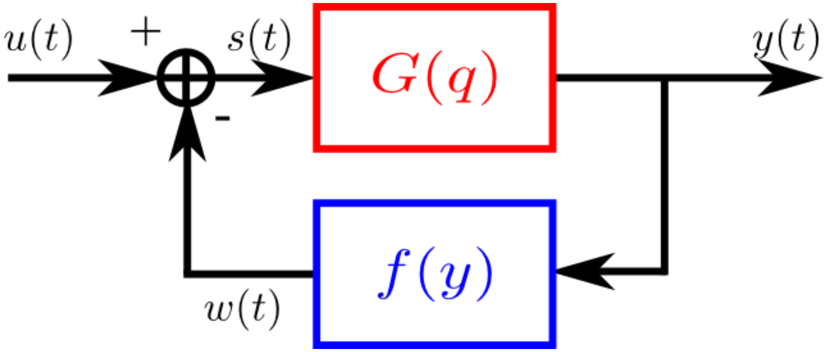
Block-Oriented Modeling?

Block-Oriented Modeling

$H(q)$ $f(x)$

Block-Oriented Modeling

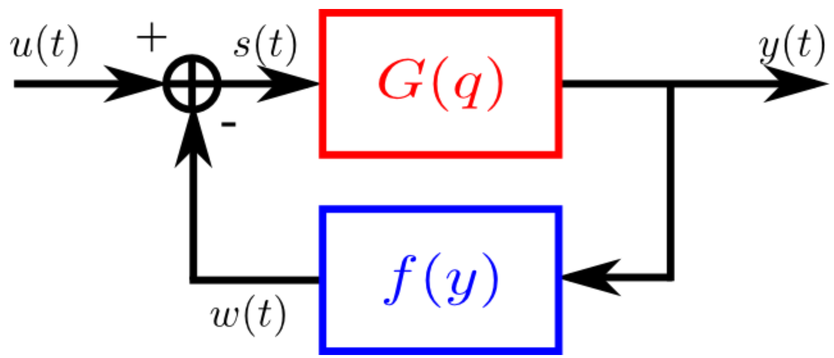




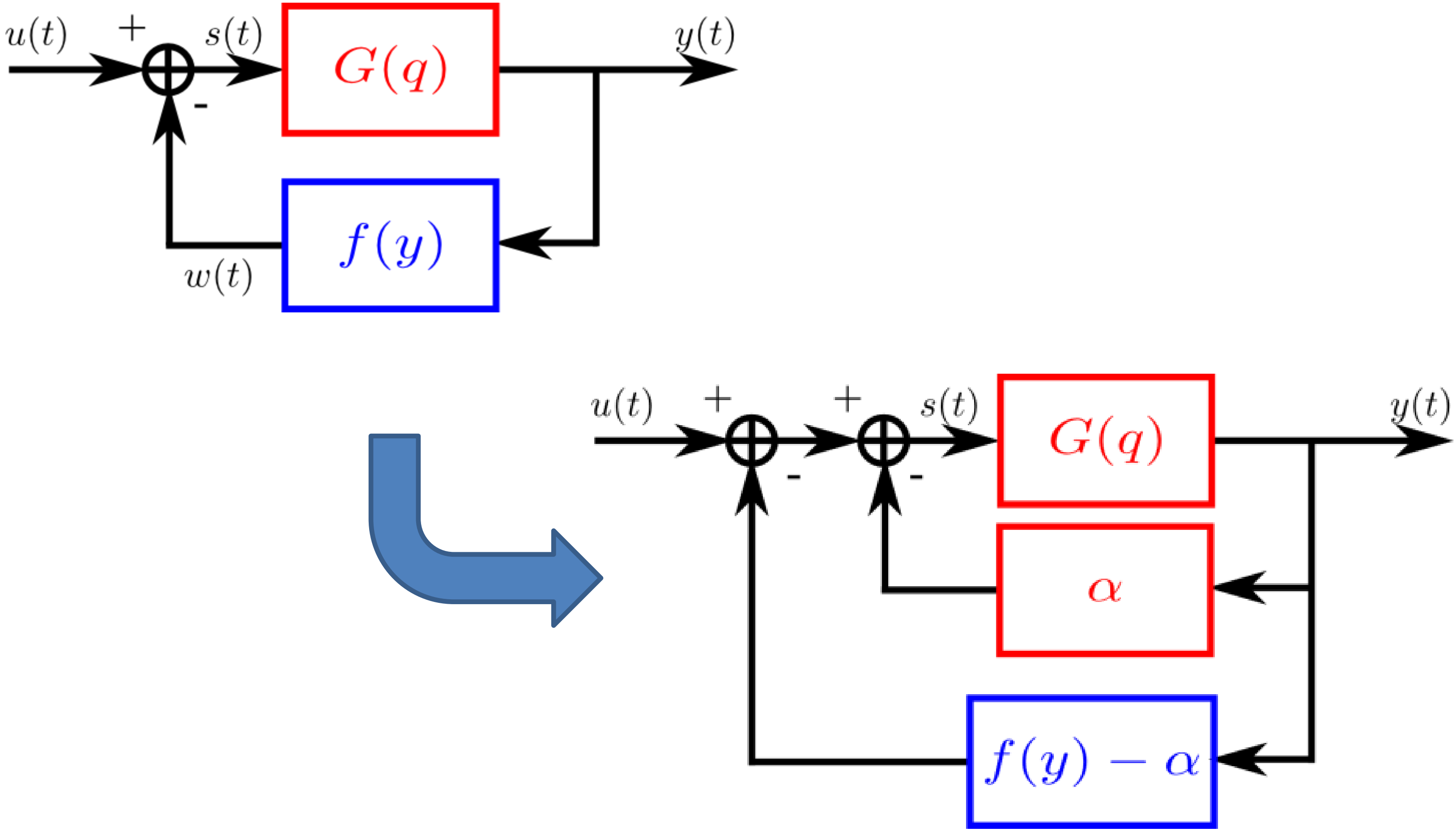
Block-Oriented Modeling

Pros	Cons
Structured	Limited flexibility
Easy to identify	Model structure selection
Easy to understand	
Easy to interpret	
Easy to analyze	
Easy to invert	

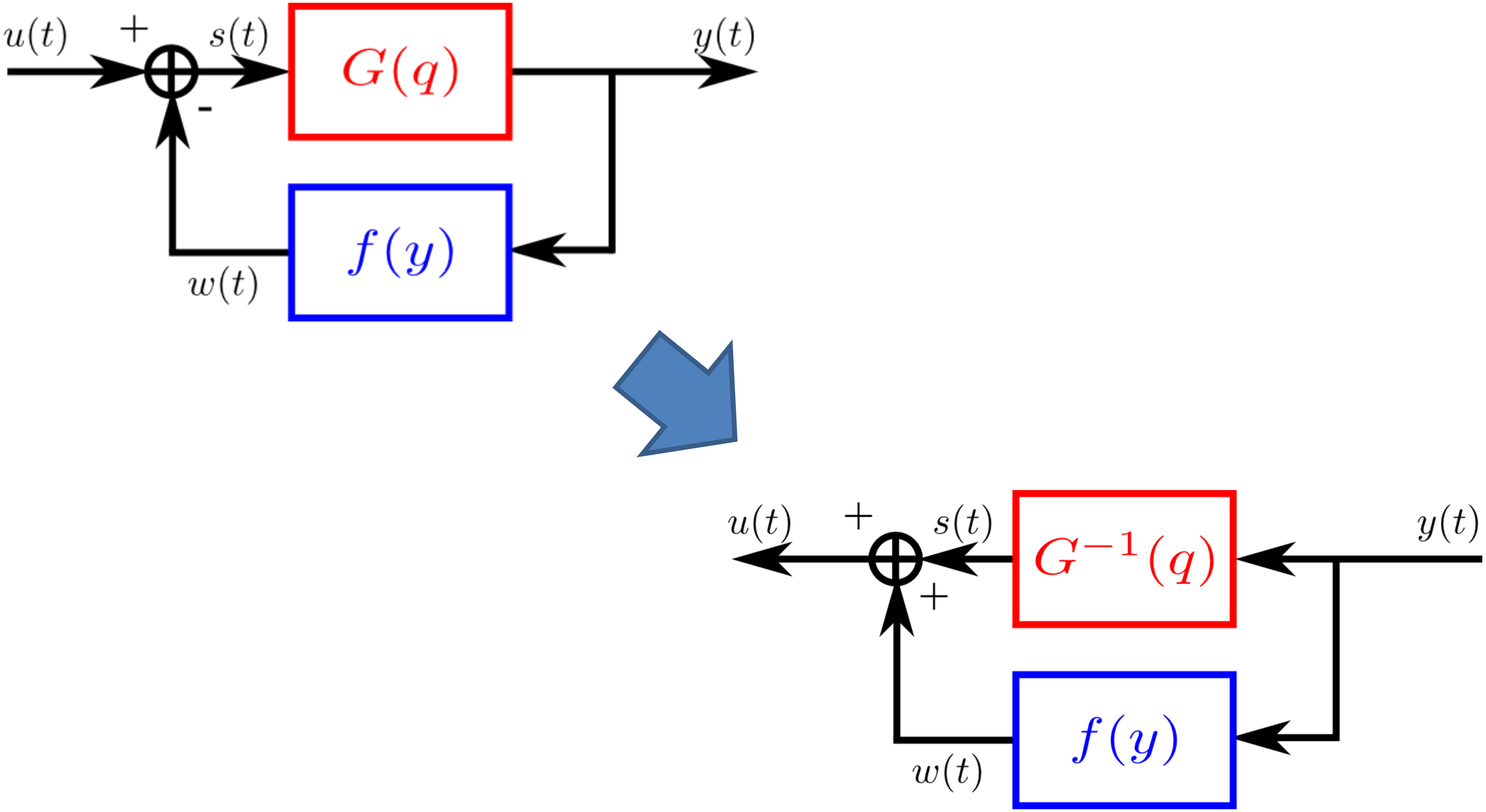
Model Structure



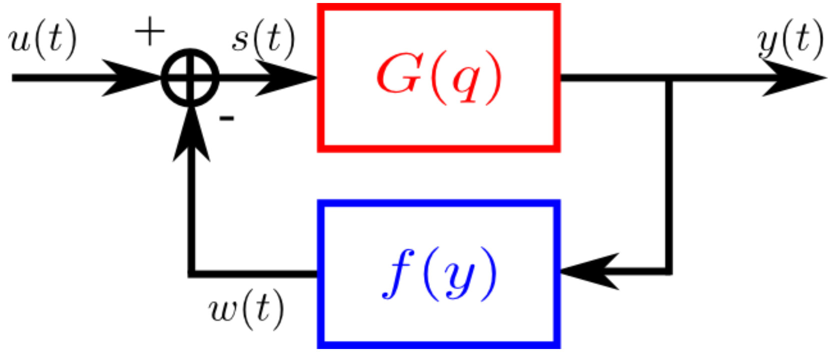
Model Structure: Identifiability



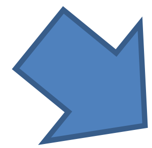
Model Structure: Inverse

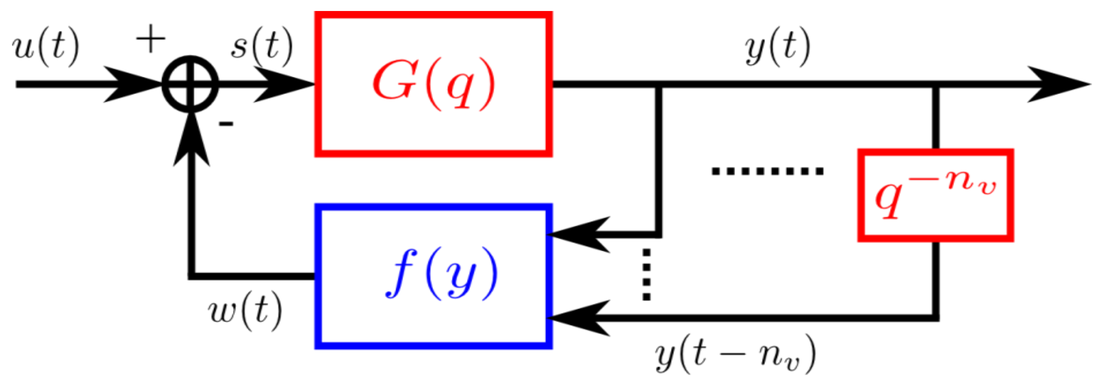


Volterra Feedback

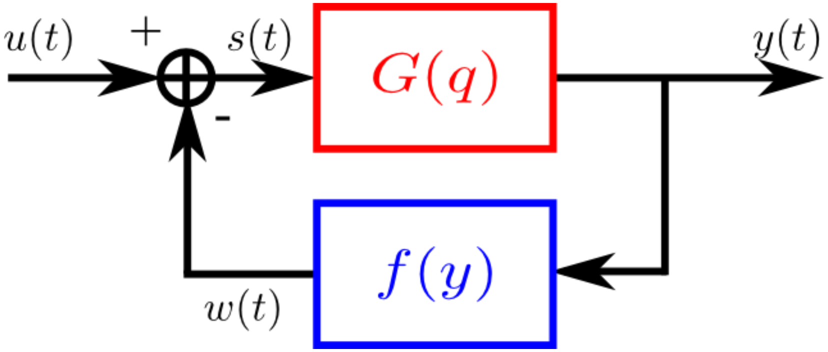


Increase Modeling Flexibility

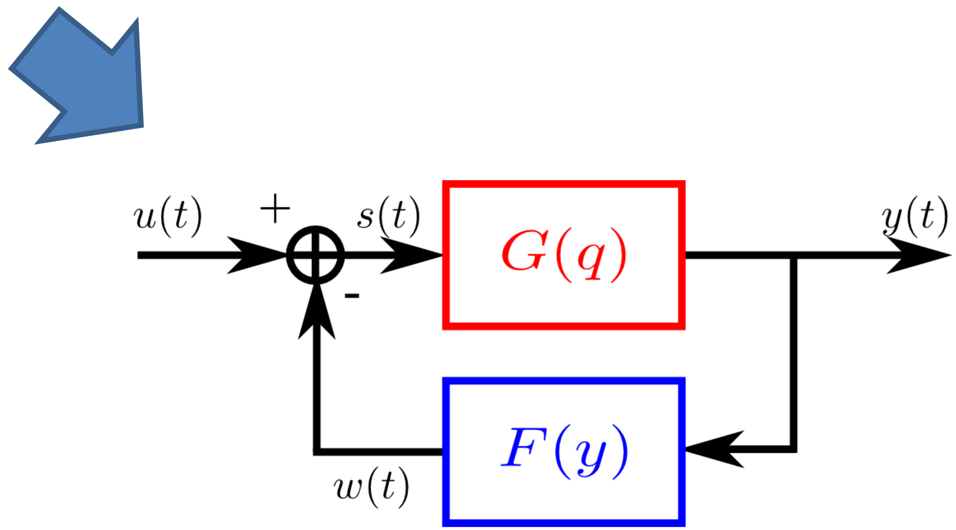




Volterra Feedback

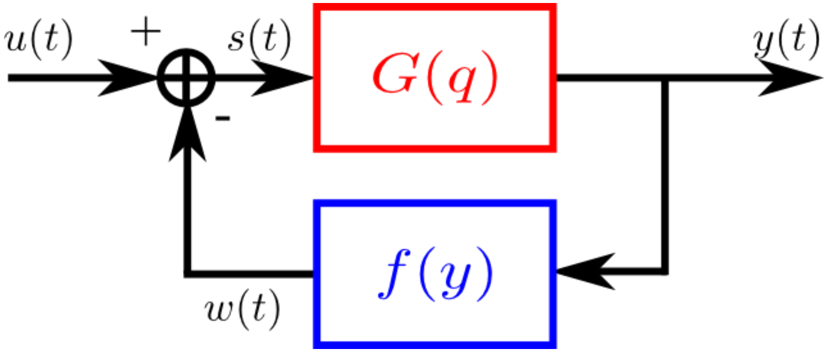


Increase Modeling Flexibility



Best Linear Approximation

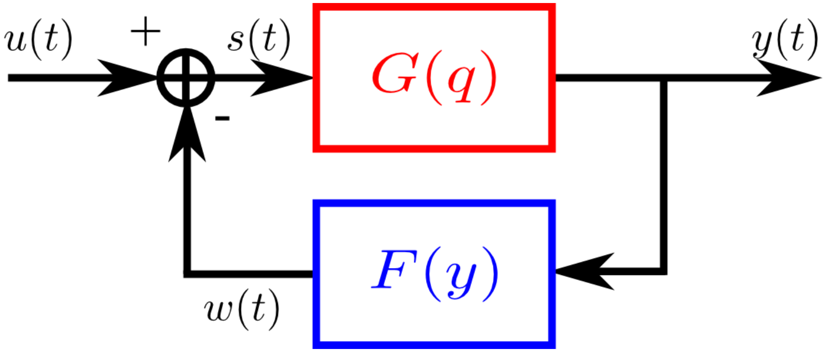
Simple Feedback Structure



$$G_{bla}(q) \approx \frac{G(q)}{1 + \gamma G(q)}$$

Best Linear Approximation

Volterra Feedback Structure



$$G_{bla}(q) \approx \frac{G(q)}{1+\gamma G(q)}$$

Assumption: Volterra dynamics are not dominant

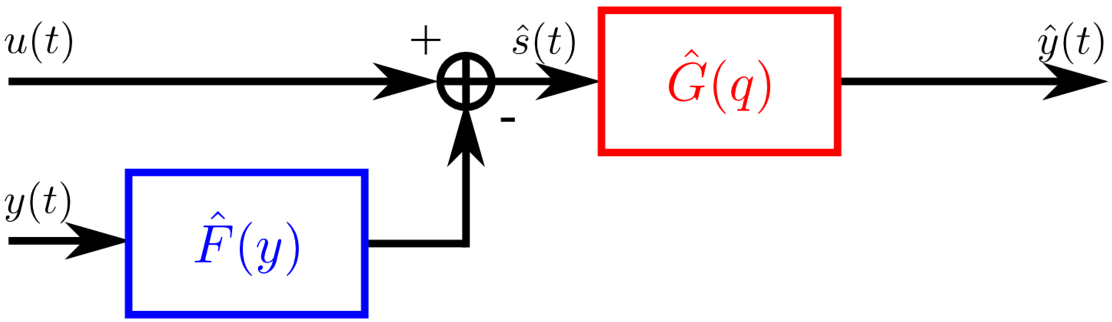
Identification

1. Estimate BLA

   at least 1-sample delay in numerator
   (avoid algebraic loops)

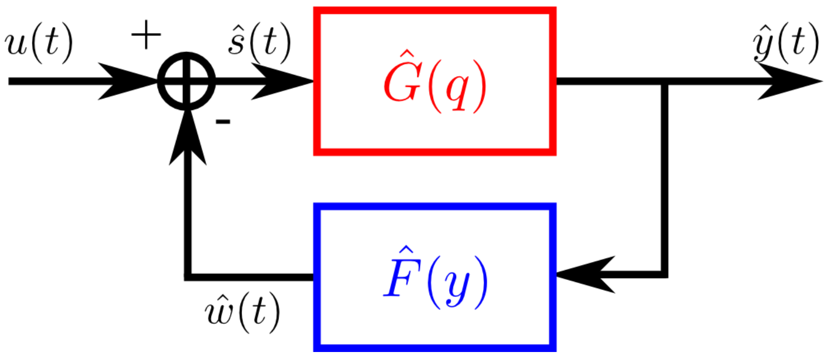
Identification

1. Estimate BLA
2. Estimate Volterra NL



Identification

1. Estimate BLA
2. Estimate Volterra NL
3. Nonlinear optimization

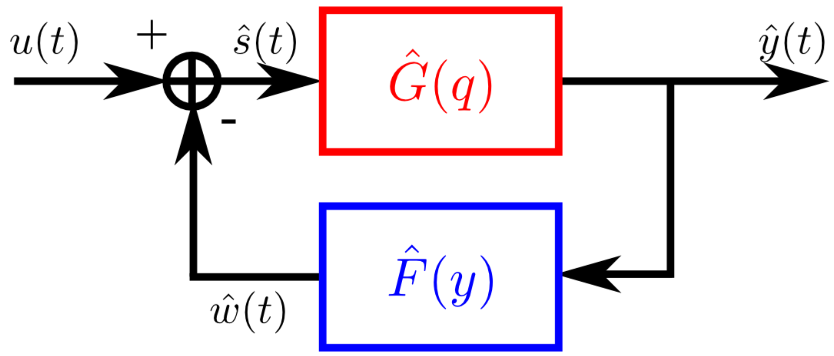


Identification – Initial Conditions

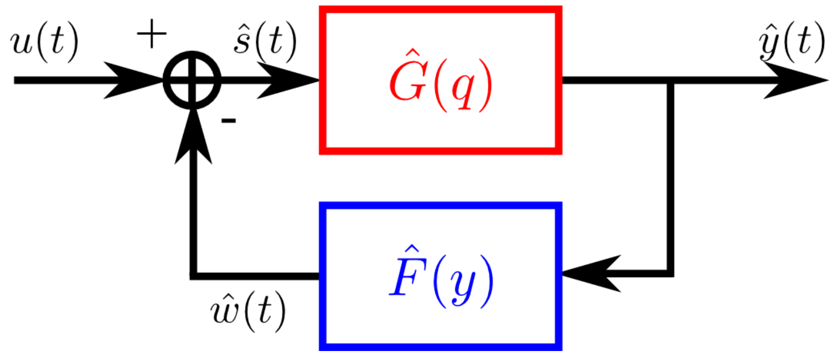
Past input and output values can be

set by user

included during optimization



Simulation/Prediction



Simulation:

Use modeled output during optimization

Prediction:

Use measured output during optimization

Results: Cascaded Tanks

BLA: order

Wiener, Hammerstein, W-H: order

Simple Feedback: order

Volterra Feedback: order

Results: Cascaded Tanks

BLA: order 1/2, 1 sample delay

Wiener, Hammerstein, W-H:

3^{rd} degree polynomial NL

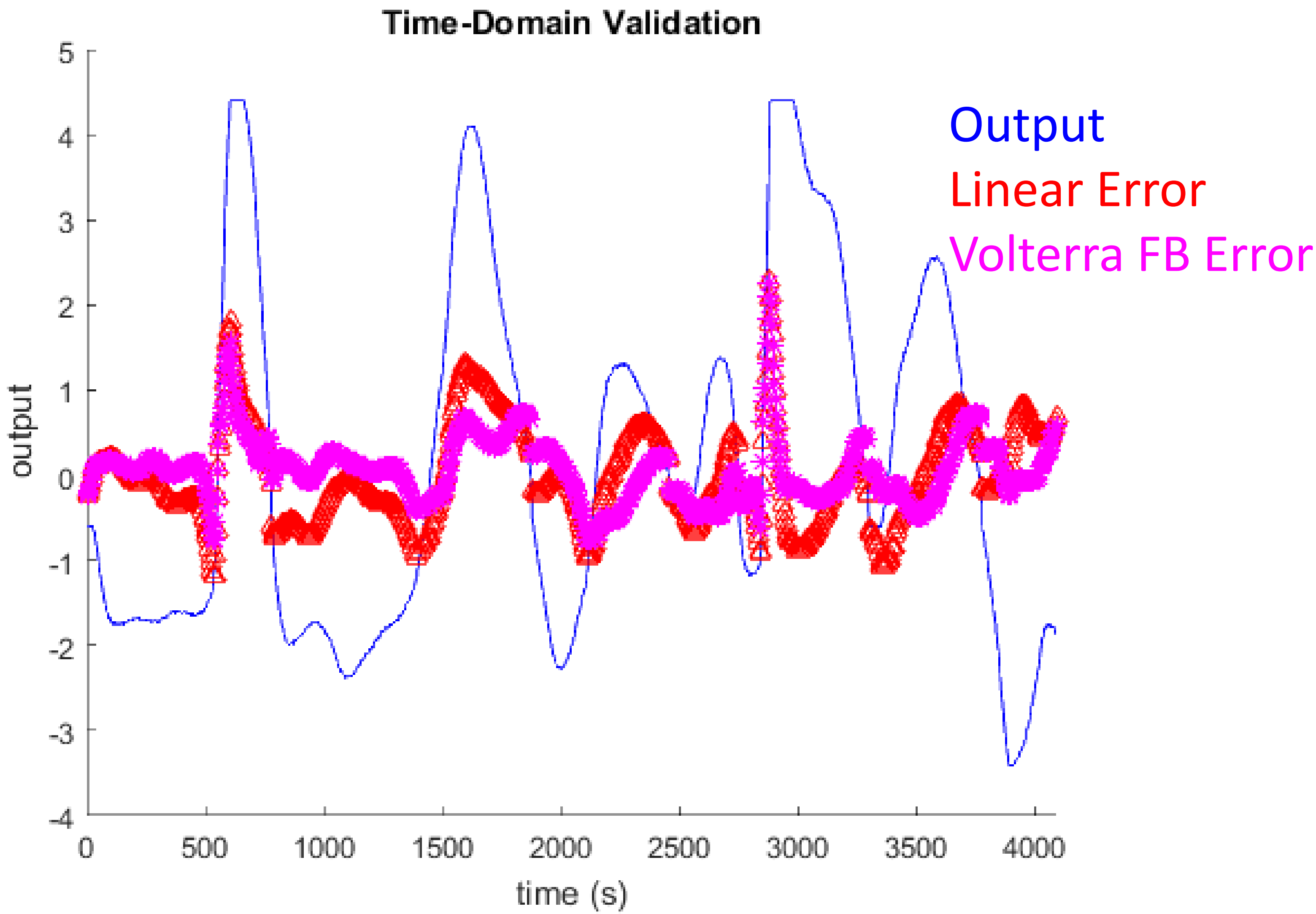
Simple Feedback:

3^{rd} degree polynomial NL

Volterra Feedback:

0 to 3^{rd} degree kernel, order 1

Results: Cascaded Tanks



Results: Cascaded Tanks

Simulation

	Estimation	Test
BLA + offset	0.5298	0.5878
Hammerstein	0.5149	0.5651
Wiener*	0.4799	0.5086
Simple Feedback	0.4316	0.4877
Volterra Feedback	0.3595	0.3972

* A Wiener structure is selected during the Wiener-Hammerstein estimation.

Results: Cascaded Tanks

Prediction

	Estimation	Test
BLA + offset	0.0484	0.0556
Simple Feedback	0.0478	0.0555
Volterra Feedback	0.0415	0.0494

Results: Bouc-Wen

Estimation Data

Random Phase Multisine Input:

- frequencies: 5-150 Hz
- RMS: 50 N
- 8192 Samples
- 2 Periods
- 10 Realizations
- fs: 750 Hz

Results: Bouc-Wen

BLA: order 2/3, 1 sample delay

Wiener, Hammerstein, W-H:

3^{rd} degree polynomial NL

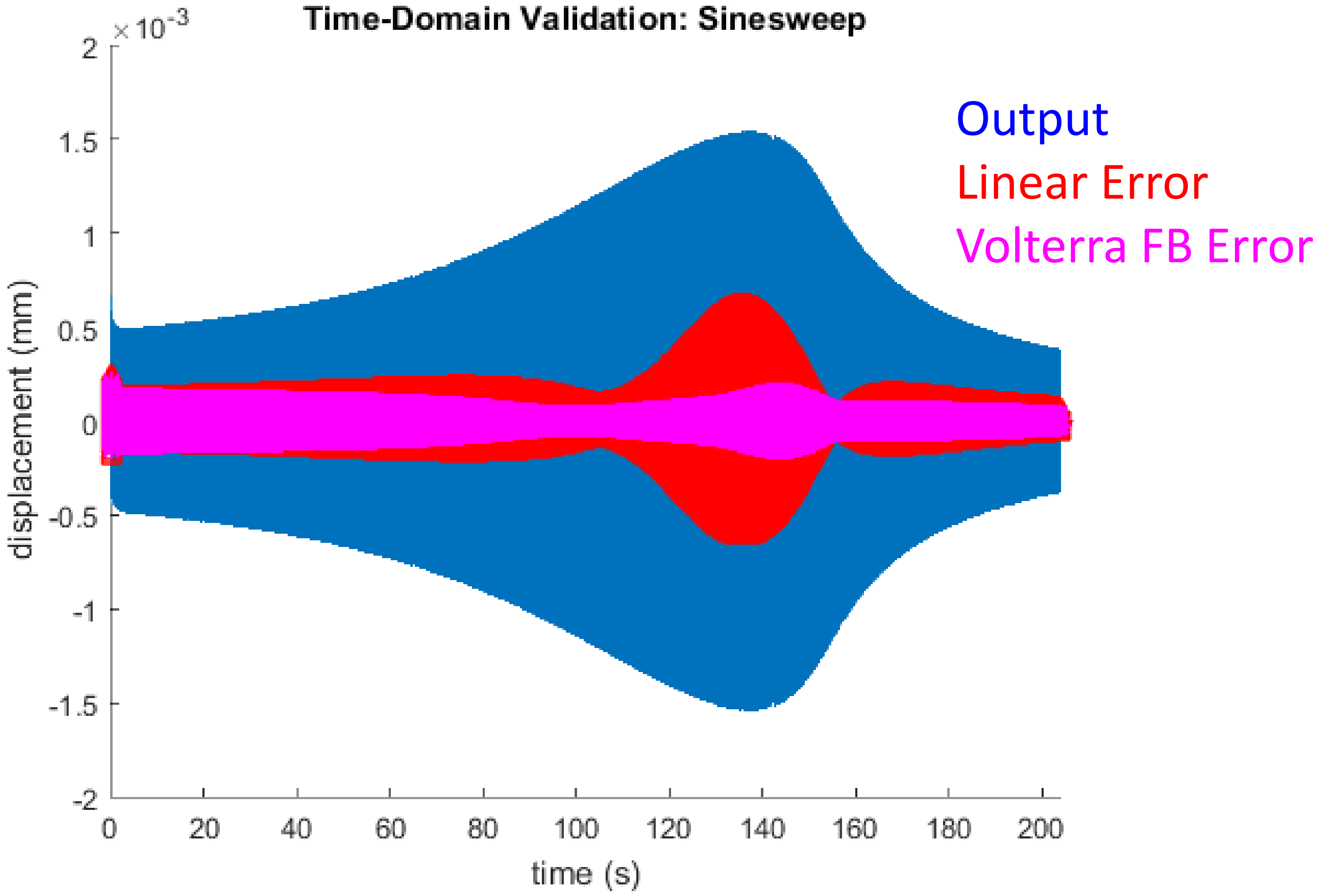
Simple Feedback:

3^{rd} degree polynomial NL

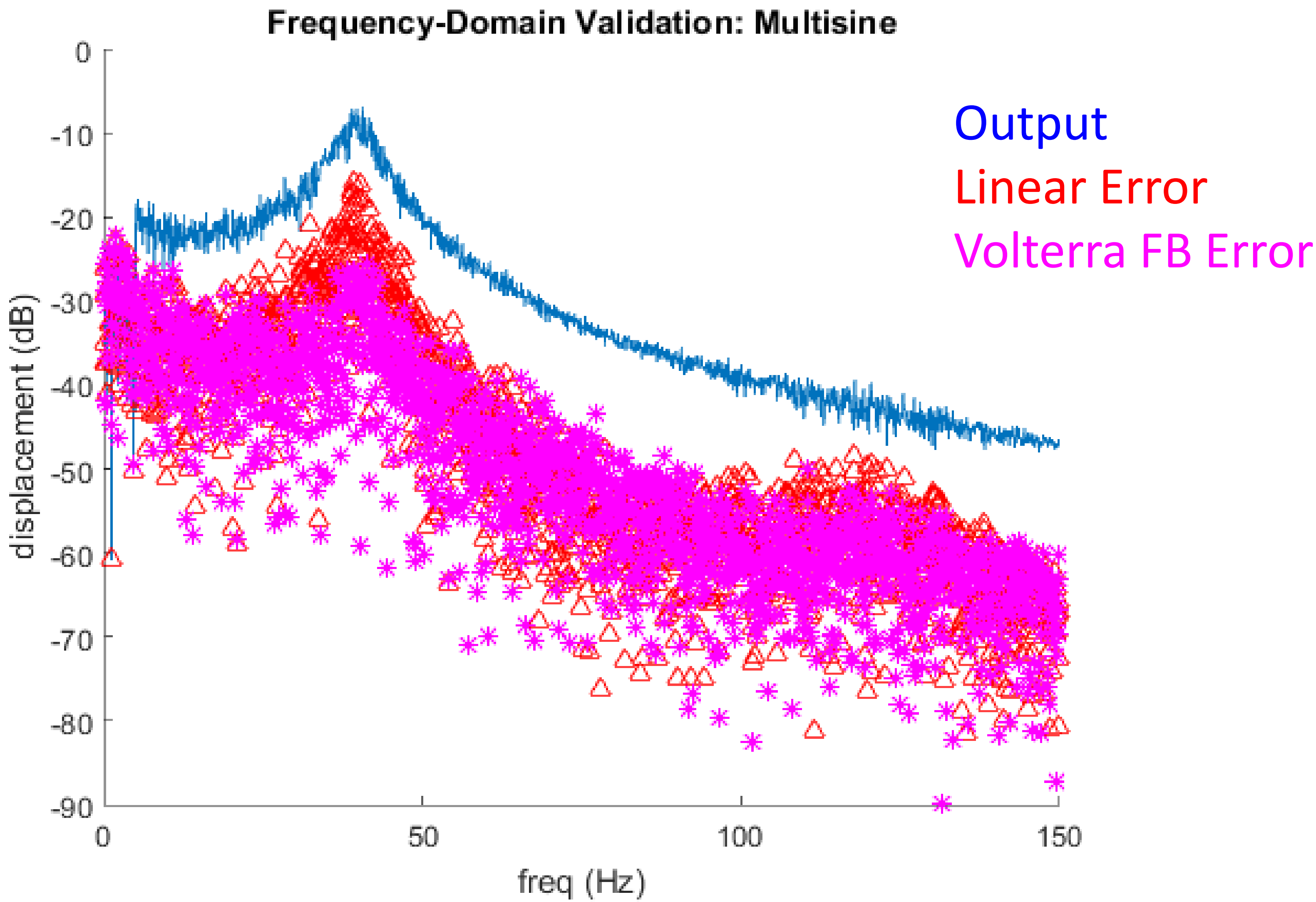
Volterra Feedback:

1^{st} and 3^{rd} degree kernel, order 1

Results: Bouc-Wen



Results: Bouc-Wen



Results: Bouc-Wen

Simulation – Validation/Test Results

	Multisine (rmse)	Sinesweep (rmse)
BLA	$15.105\ 10^{-5}$	$16.619\ 10^{-5}$
Wiener	$14.877\ 10^{-5}$	$16.235\ 10^{-5}$
Hammerstein	$14.967\ 10^{-5}$	$18.691\ 10^{-5}$
Wiener-Hammerstein	$14.875\ 10^{-5}$	$16.224\ 10^{-5}$
Simple Feedback	$12.091\ 10^{-5}$	$15.004\ 10^{-5}$
Volterra Feedback	$8.755\ 10^{-5}$	$6.392\ 10^{-5}$

Results: Bouc-Wen

Prediction – Validation/Test Results

	Multisine (rmse)	Sinesweep (rmse)
BLA	$1.126\ 10^{-5}$	$0.698\ 10^{-5}$
Simple Feedback	$0.915\ 10^{-5}$	$0.451\ 10^{-5}$
Volterra Feedback	$0.895\ 10^{-5}$	$0.347\ 10^{-5}$

Conclusions

Volterra Feedback:

- More flexible model structure
- Easy to invert
- Simple identification algorithm
- Good results

But:

- Still large model errors (e.g. hysteresis)