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Structural modeling of Wiener-Hammerstein system in the presence of the process noise

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Problem statement

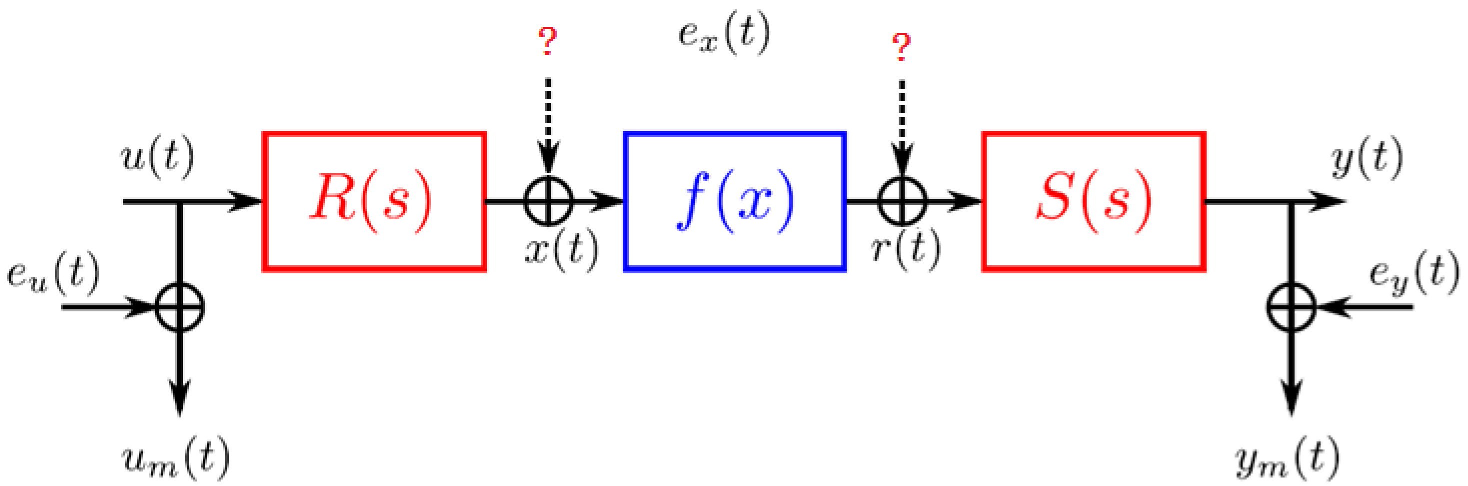


Fig. 1 Wiener-Hammerstein system

- **Process noise modeling influences the estimation consistency of Wiener-Hammerstein model**
- **Detect whether the process noise passes through the static nonlinear part**

Assumptions

- **Wiener-Hammerstein model**

(1) The systems $R(q)$ and $S(q)$ are linear, stable and time-invariant

(2) The static nonlinearity $f(t)$ belongs to the set of generalized nonlinearities, can be approximated arbitrary well by polynomials in the sense that the mean square error tends to zero as the polynomial degree tends to 1.

- **Measurement and process noise**

The output measurement noise $e_y(t)$ and the process noise $e_x(t)$ are zero-mean *stationary* noise, and independent of the input excitation.

- **System input**

The input signal $u(t)$ is a persistent excitation, whose value is bounded.

Assumptions

Static nonlinearity:

$$f(x) = \sum_{i=0}^{\infty} a_i x^i$$

(Weierstrass approximation theorem)

Approximately,

$$f(x) = \sum_{i=0}^{n_f} a_i x^i$$

System response

- The process noise does pass through the nonlinearity part (see Fig. 1)

$$y_m(t) = \underbrace{\sum_{i=0}^{\infty} a_i S(q) x^i(t)}_{\text{nonlinear system response}} + \underbrace{\sum_{i=1}^{\infty}\sum_{j=0}^{i-1} a_i c_{ij} S(q)\left\{x^j(t) e_x^{i-j}(t)\right\}}_{\text{error due to process noise}} + \underbrace{e_y(t)}_{\text{measurement noise}}$$

where $x(t) = R(q)u(t)$,

$$c_{ij} = \frac{i!}{(i-j)!j!} \quad \text{(binomial coefficient)}$$

Remark: the error caused by the process noise is "modulated" by the input signal!

System response

- **The process noise does not pass through the nonlinearity part**

$$y_m(t) = \underbrace{\sum_{i=0}^{\infty} a_i S(q) x^i(t)}_{\text{nonlinear system response}} + \underbrace{S(q) e_x(t)}_{\text{error due to process noise}} + \underbrace{e_y(t)}_{\text{measurement noise}}$$

where $x(t) = R(q)u(t)$.

Remark: the error caused by the process noise is independent of the input signal!

Process noise detection

Principle

- Use the stationarity of the ouput measurement noise
- Use nonstationary input signal to differentiate the output error caused by the process noise from the measurement noise when it passes through the nonlinearity part
- Use the periodic signal to separate the nonperiodic part (caused by the process noise and measurement noise) from the nonlinearity of Wiener-Hammerstein system

Design a periodic signal which is nonstationary within one period

Process noise detection

Input design

$$u(t) = 1/\sqrt{N} \sum_{k=-N/2+1}^{N/2-1} |U(k)| \exp\left(j2\pi \frac{k f_s}{N} t + \phi_k\right)$$ **Random phase multisine**

with $\mathbb{E}\left(\exp(j\phi_k)\right) = 0$

Iterative optimization with ϕ_k as design variables

(1) At the *i*+1 iteration step

$$u_{i+1}(t) = \frac{RMS_0(t)}{RMS_i(t)} u_i(t)$$

***RMS_0*: expected enveloppe**

***RMS_i*: iterated enveloppe at the *i*-step**

(2) Compute the DFT

$$U_{i+1}(k) = A_{i+1}(k) \exp(j\phi_k)$$

(3) Impose constraint

$$\tilde{U}_{i+1}(k) = \begin{cases} A \exp(j\phi_k) & k \in \mathcal{I} \\ 0 & k \notin \mathcal{I} \end{cases}$$

Process noise detection

The output error $e_p(t)$ caused by the process noise

$$e_p(t) = \sum_{i=1}^{n_f} \sum_{j=0}^{i-1} a_i c_{ij} S(q) \left\{ x^j(t) e_x^{i-j}(t) \right\}$$ **(Preceding the SNL part)**

$$e_p(t) = S(q) e_x(t)$$ **(Succeeding the SNL part)**

Bounded property

$e_p(t)$ is aleatory, and has finite mean-value and variance at each time instant

Process noise detection

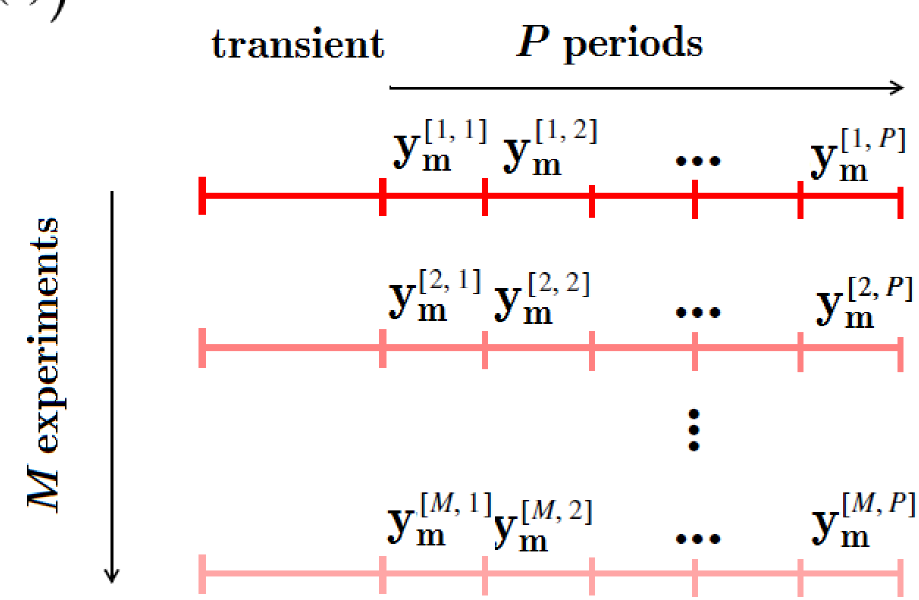
Robust measurement strategy using multiple experiments

$$\hat{y}^{[m]}(t) \;=\; \tfrac{1}{P}\sum_{p=1}^{P} y^{[p,m]}$$

$$\hat{\sigma}^2_{e[m]}(t) = \frac{1}{P-1}\sum_{p=1}^{P}\left(y^{[p,m]}(t) - \hat{y}^{[m]}(t)\right)^2$$

$$\hat{\sigma}^2_{e}(t) = \frac{1}{M}\sum_{m=1}^{M}\hat{\sigma}^2_{e[m]}(t)$$

***M* experiments, *P* periods**



Simulated example

Wiener-Hammerstein system

Linear systems

$$R(z) = \frac{0.1 + 0.2z^{-1} - 0.3z^{-2}}{0.95 - 1.4z^{-1} + 0.9z^{-2}}$$

$$S(z) = \frac{z^{-1} + 0.5z^{-2}}{0.95 - 0.9z^{-1} + 0.9z^{-2}}$$

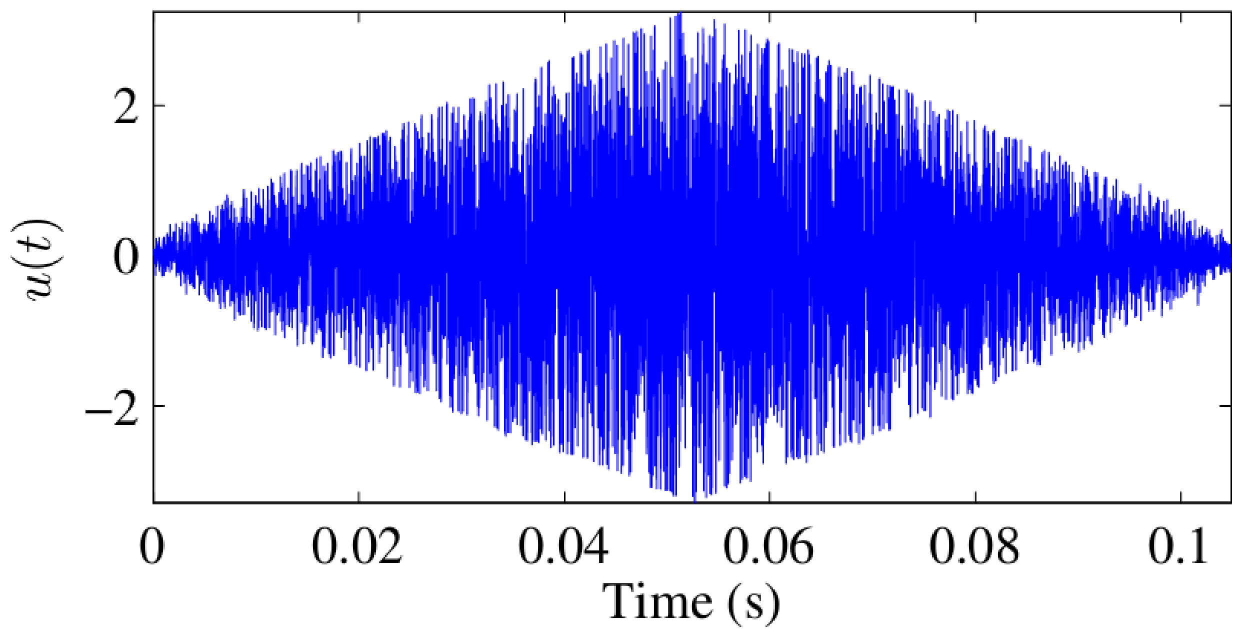
Static nonlinearity (SNL)

$$f(x(t)) = \alpha x(t) + \beta x^2(t) + \gamma x^3(t)$$

where $\alpha = 0.01,\ \beta = 0.05,\ \gamma = -0.008$

Simulated example

Input signal (one period)



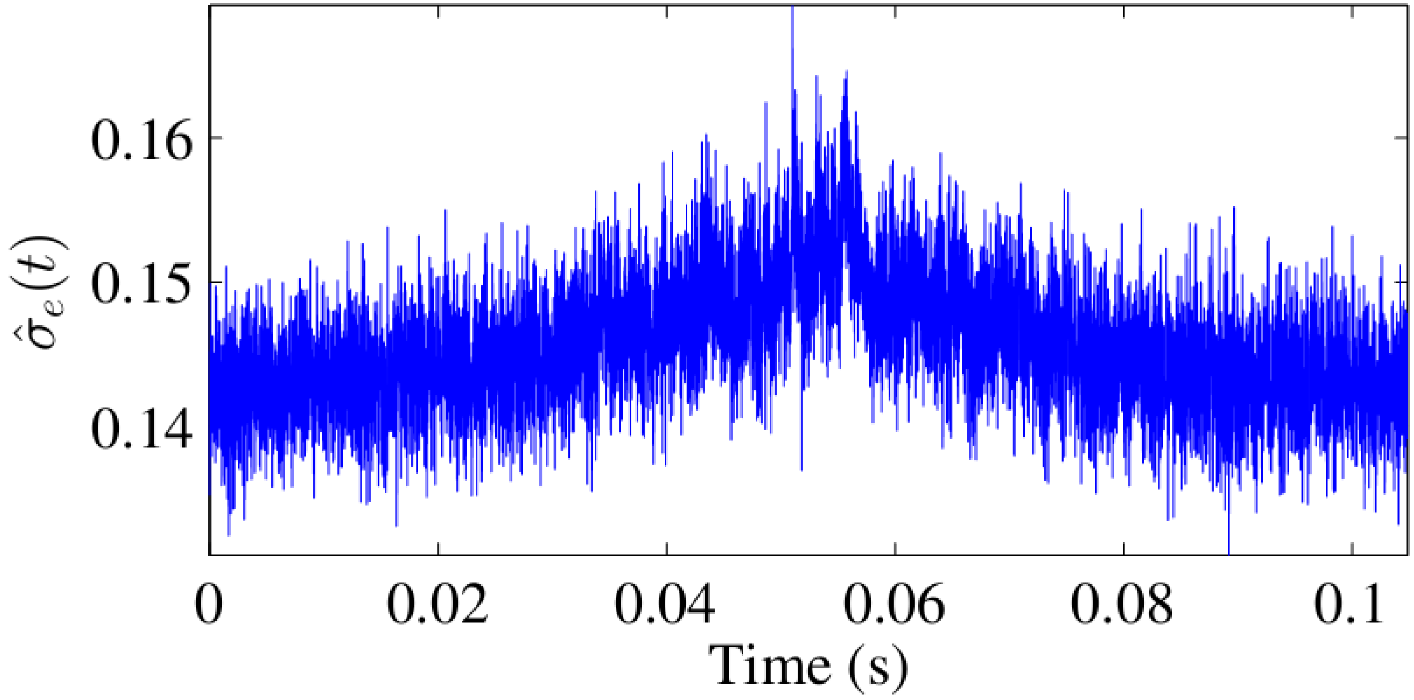
Sample number *N* = 8192, 2132 excited frequency lines

Periods of signals: *P* =32, experiments: *M* = 32

Simulated example

Wiener-Hammerstein system: heavy output measurement noise

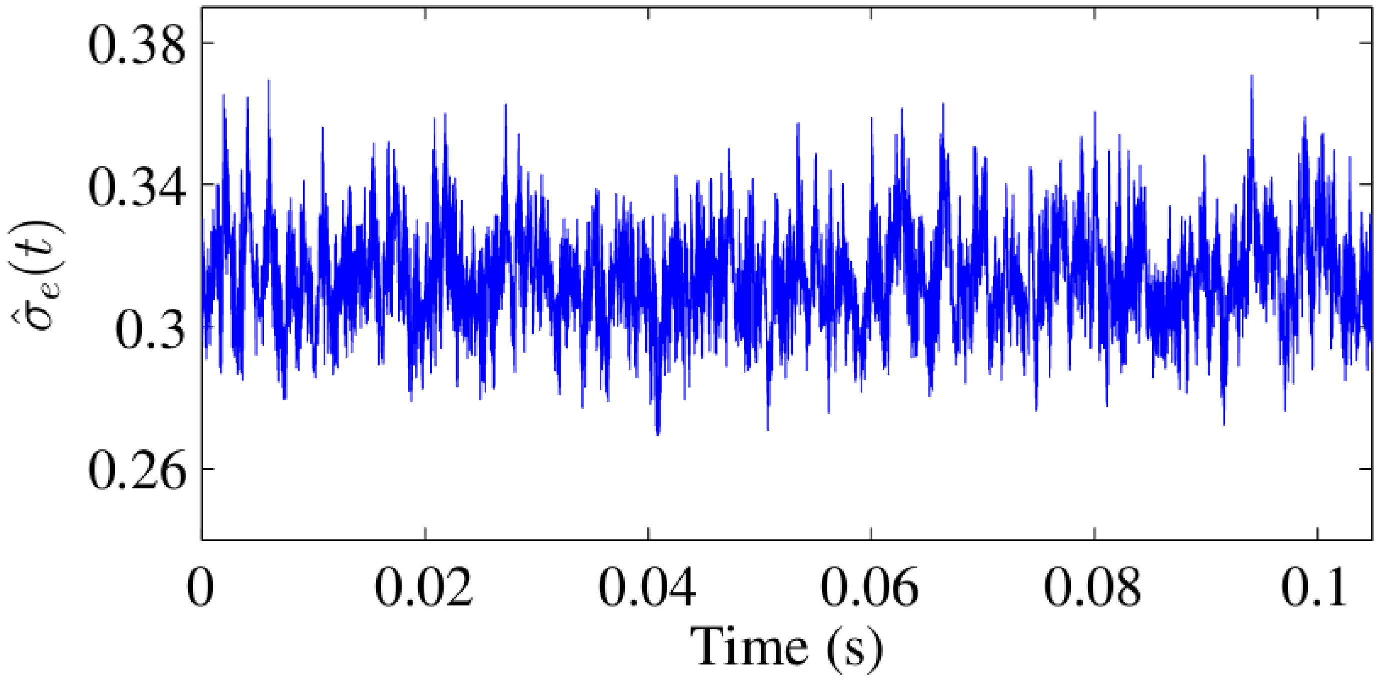
Preceding the SNL part



Simulated example

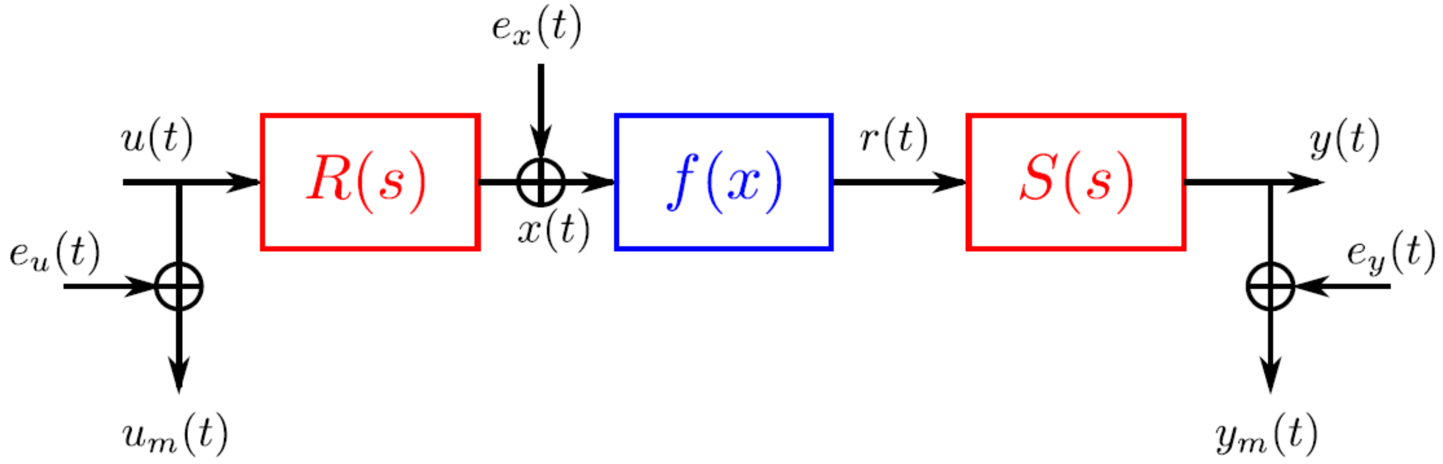
Wiener-Hammerstein system: heavy process noise

Succeeding the NL part



Wiener-Hammerstein benchmark

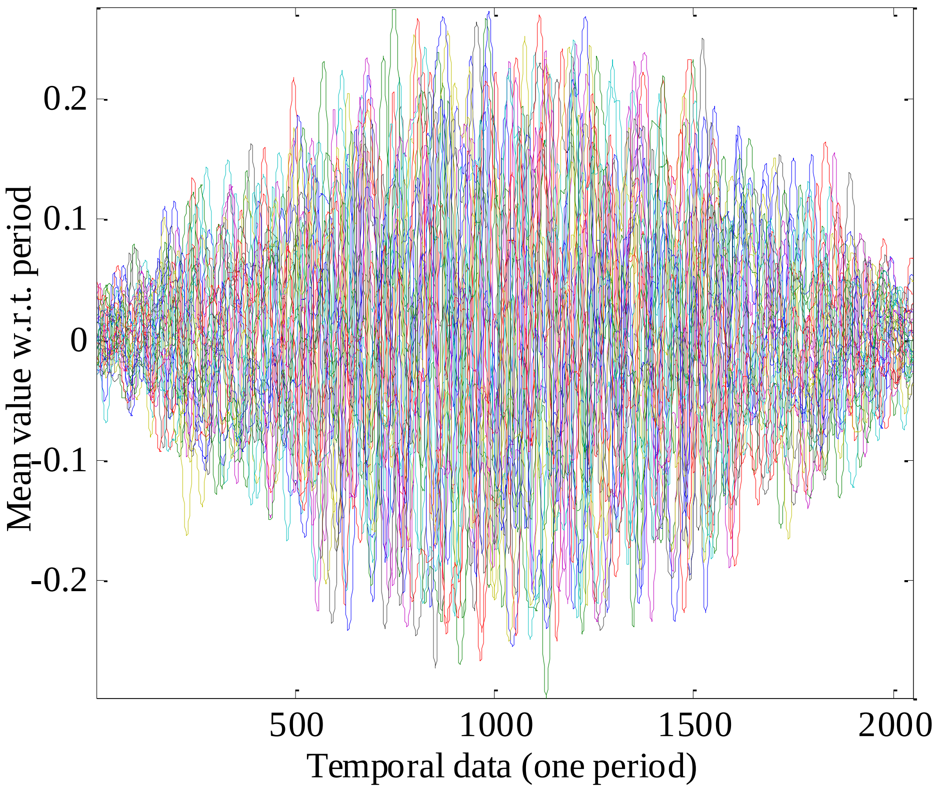
Wiener-Hammerstein benchmark with process noise



Periods of signals: *P* = 30
Experiments: *M* = 32

Wiener-Hammerstein benchmark

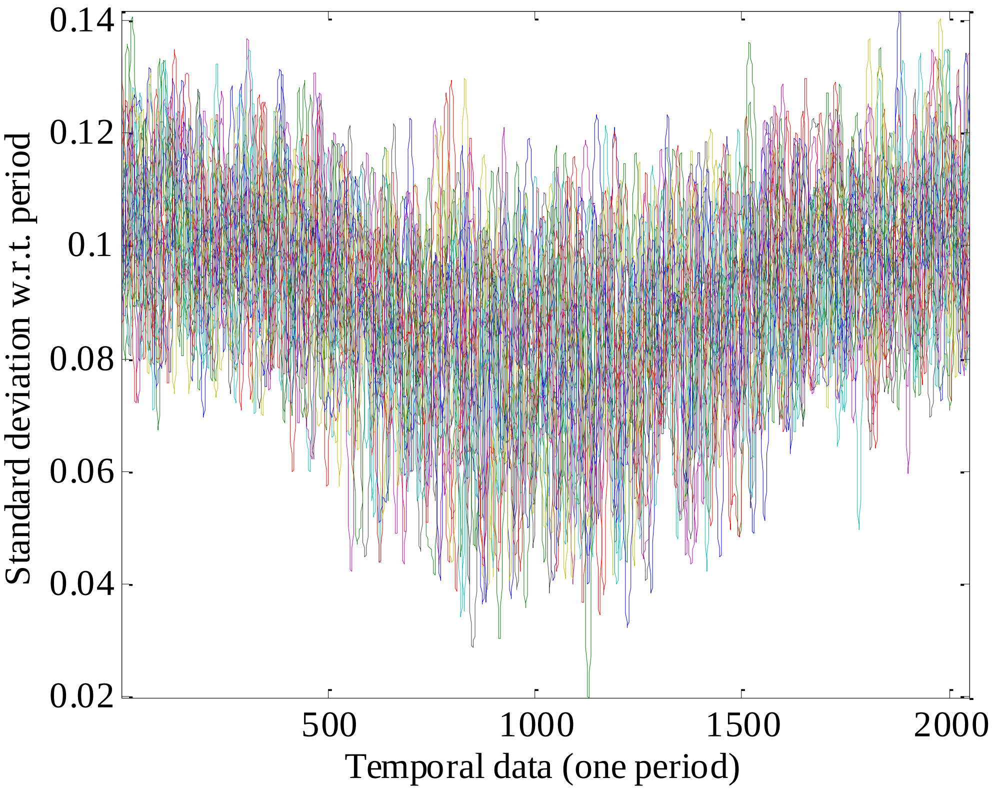
Averaged system response over periods for 32 experiments



$\widehat{y}_m^{[m]}(t)$

Wiener-Hammerstein benchmark

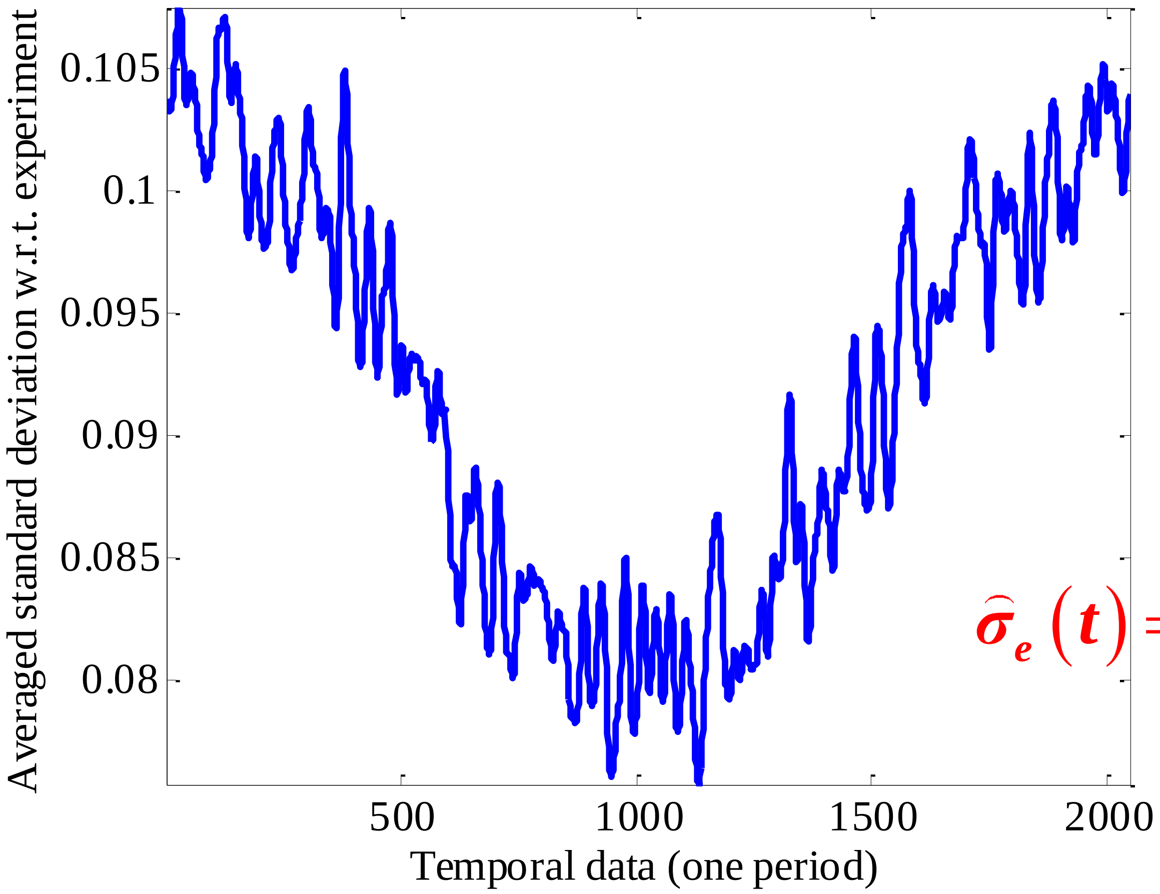
Standard deviations of output error, including measurement noise + error caused by the process noise, for 32 experiments



$\widehat{\sigma}_{e^{[m]}}(t)$

Wiener-Hammerstein benchmark

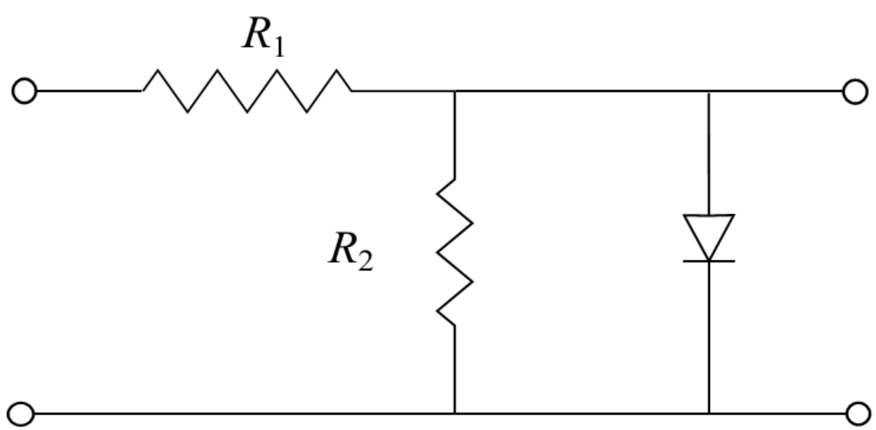
Standard deviation of output error over all experiments



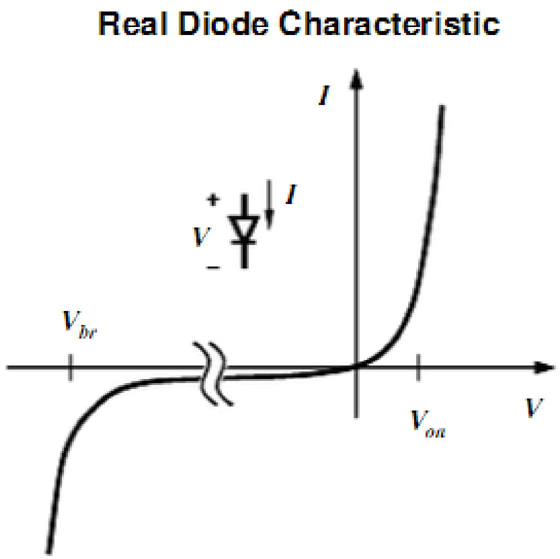
$$\hat{\sigma}_e(t) = \frac{1}{M-1} \sum_{m=1}^{M} \hat{\sigma}_{e^{[m]}}(t)$$

Wiener-Hammerstein benchmark

Physical interpretation of decreasing-increasing behavior of the estimated output error



Circuit generated the static nonlinearity



Conclusion

- **A simple framework has been proposed for the structural modeling of Wiener-Hammerstein systems with process noise.**
- **The proposed methodology can provide insight on the static nonlinearity which the process noise precedes.**
- **It can be straightforwardly applied to other block-oriented models.**