Identification of Nonlinear Mechanical Systems: State of the Art and Recent Trends

Gaëtan Kerschen
Space Structures and Systems Laboratory
Aerospace and Mechanical Eng. Dept.
University of Liège

Colleagues and Collaborators:
J.P. Noël, J. Schoukens, K. Worden, B. Peeters
Why Is NSI Important in Mechanical Engineering?

Harmonic and Random Nonlinear Vibration Test and Simulation of an aircraft piccolo tube

Contact nonlinearities

→ 300% error between predictions and measurements
Why Is NSI Important in Mechanical Engineering?

Introduction of two poles around one nonlinear mode

→ Linear tools and methods may fail
Outline

A brief review of sources of nonlinearity in mechanics.

State-of-the-art: seven families of NSI methods in mechanical engineering.

Future trends, including identification for design.
3 Main Assumptions for Linear Mechanical Systems

\[ M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = f(t) \]

- **Linear elasticity** → nonlinear materials
- **Small displ. and rotations** → geometric nonlinearity
  → nonlinear boundary conditions
- **Viscous damping** → nonlinear damping mechanisms
Source 1: Material Nonlinearities

Hyperelastic material (e.g., rubber)

Shape memory alloy
Anti-Vibration Mounts of a Helicopter Cockpit

Decrease in stiffness
Decrease in damping

A. Carrella, IJMS 2012
\[ \ddot{\theta} + \omega_0^2 \sin \theta = 0 \]

\[ \sin \theta = \theta - \frac{\theta^3}{6} + \ldots \]

\[ F = 2k (l_1 - l_0) \frac{x}{l_1} = 2kx \left( 1 - \frac{l_0}{\sqrt{x^2 + l_0^2}} \right) \]

\[ \frac{l_0}{\sqrt{x^2 + l_0^2}} = 1 - \frac{x^2}{2l_0^2} + \frac{3x^4}{8l_0^4} + O(x^6) \]
A Widely-Used Benchmark in Mech. Engineering

Beam @ ULg

Increase in stiffness
The SmallSat Spacecraft

Goals
- Micro-vibration mitigation
- Large amplitude limitation

Solutions
- Elastomer plots
- Mechanical stops

SmallSat spacecraft (EADS Astrium)

NL isolation device for reaction wheels

Elastomer plots

Mechanical stops
Contact Can Generate Complex Dynamics

Dangerous nonlinear resonance

Accel. (m/s²) 0

Sweep frequency (Hz)
Nonlinear damping, a pleonasm!

Extremely complex.

Present in virtually all interfaces between components (e.g., bolted joints).

Key parameter, because it dictates the response amplitude.

Bouc-Wen benchmark.
Sliding Connection in the F-16 Aircraft
Impact of the Connection on the F-16 Dynamics

Decrease in stiffness
Increase in damping

FRF

Frequency (Hz)
A brief review of sources of nonlinearity in mechanics.

State-of-the-art: seven families of NSI methods in mechanical engineering.

Future trends, including identification for design.
A Three-Step Process

Yes or No?

What? Where? How?

Yes or No?

What? Where? How?

More information but increasingly difficult

1. Detection

2. Characterization

3. Parameter estimation

\[ f_{nl}(x, \dot{x}) = x^3, \sin x, |\dot{x}| \]

\[ f_{nl}(x, \dot{x}) = 0.1x^3, 1.2x^3, -3x^3 \]
Three Main Differences

<table>
<thead>
<tr>
<th>MECHANICS</th>
<th>EE/CONTROL</th>
</tr>
</thead>
<tbody>
<tr>
<td>The quantification of the impact of nonlinearity is not often performed.</td>
<td>Best-linear approximation (Schoukens et al.).</td>
</tr>
<tr>
<td>White-box approach commonplace.</td>
<td>The black-box approach seems to be very popular.</td>
</tr>
<tr>
<td>It is only very recently that uncertainty is accounted for (e.g. Beck, Worden et al.).</td>
<td>1965 (!): Astrom and Bohlin introduced the maximum likelihood framework.</td>
</tr>
</tbody>
</table>

*A reputable engineer should never deliver a model without a statement about its error margins (M. Gevers, IEEE Control Systems Magazine, 2006)*
Often, reasonably accurate low-dimensional models can be obtained from first principles.
But Not Always: Individualistic Nature of Nonlinearities

Elastic NL (grey-box)

Damping NL (black-box or further analysis)
Case Study: The F-16 Aircraft (With VUB & Siemens)

Right missile

Connection with the wing
Apply Your Favorite Modal Analysis Software

FRF

Introduction of two poles around one nonlinear mode

6.5  7.5  Frequency (Hz)

F-16
Nonlinearity can often be seen in raw acceleration signals.
Measured Frequency Response Functions

At this stage, we should be convinced about the presence of nonlinearity.
Time-Frequency Analysis (Wavelet Transform)

Objective: know more about the nonlinear distortions.
Restoring Force Surface Method (Masri & Caughey)

Objective: visualize nonlinearities.

Nonlinear connection instrumented on both sides.
Restoring Force Surface Method
A Three-Step Process

Yes or No?

What? Where? How?

Yes or No?

What? Where? How?

1. Detection

2. Characterization

3. Parameter estimation

More information but increasingly difficult

\[ f_{nl}(x, \dot{x}) = x^3, \sin x, |\dot{x}| \]

\[ f_{nl}(x, \dot{x}) = 0.1x^3, 1.2x^3, -3x^3 \]
Classification in Seven Families

1. By-passing nonlinearity: linearization
2. Time-domain methods
3. Frequency-domain methods
4. Time-frequency analysis
5. Black-box modeling
6. Modal methods
7. Finite element model updating

Time- and Frequency-Domain Methods

Often manipulation of equations of motion giving rise to a least-squares estimation problem (restoring force surface, conditioned reverse path, generalizations of subspace identification methods)

The Volterra and higher-order FRFs theories are popular within our community, but have never found application on realistic structures.
Time-Frequency Analysis

Hilbert transform and its generalization to multicomponent signals (empirical mode decomposition).

Wavelet transform.
Interesting when there is no a priori knowledge about the nonlinearity.

But...

A priori information and physics-based models should not be superseded by any ‘blind’ methodology.

Overfitting may be an issue.

Characterized by many parameters; difficult to optimize.
Modal Methods

Different approaches exist in the literature:

- *linearised modes*
  
  intuitive, but do not account for NL phenomena.

- *data-based modes*
  
  straightforward, but limited theoretical background.

- *nonlinear normal modes*
  
  rigorous, and do fully account for NL phenomena.
Finite Element Model Updating

FE model

Experimental data

Feature extraction

???

Model updating

Correlation

Parameter selection

O.F. min.

Good

Reliable model

Poor
Where Do We Stand?


First contributions Focus on 1DOF: Hilbert, Volterra Focus on MDOF: NARMAX, frequency-domain ID, finite element model updating Large-scale structures with localized nonlinearities: uncertainty quantification, extension of linear algorithms (nonlinear subspace ID)
Identification for Design

Computer-aided modelling (FEM, …)

MEASURE ➔ IDENTIFY ➔ MODEL ➔ UNDERSTUD UNCOVER ➔ DESIGN

F-16 aircraft

SmallSat spacecraft
The SmallSat Spacecraft

SmallSat spacecraft (EADS Astrium)

NL isolation device for reaction wheels

Elastomer plots

<table>
<thead>
<tr>
<th>Goals</th>
<th>Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Micro-vibration</td>
<td>Elastomer plots</td>
</tr>
<tr>
<td>mitigation</td>
<td></td>
</tr>
<tr>
<td>Large amplitude</td>
<td>Mechanical stops</td>
</tr>
<tr>
<td>limitation</td>
<td></td>
</tr>
</tbody>
</table>
Troubleshooting Clearly Needed
24 Accels Close to the Suspected Nonlinear Device
Wavelet Transform: Nonsmooth Nonlinearity

![Wavelet Transform Figure]

- Instantaneous frequency (Hz)
- Sweep frequency (Hz)

Key steps:
- Measure
- Identify
- Model
- Understand
- Uncover
- Design
Time Series: Clearance Identification

Relative displacement (−)

Sweep frequency (Hz)

Jump

Discontinuity
Acceleration Surface: Nonlinearity Visualization

- Acc.

0.6 g

Rel. displ.

1 g

- Acc.

Rel. displ.

MEASURE

IDENTIFY

MODEL

UNDERSTD UNCOVER

DESIGN
Experimental Model of the Nonlinearity

Fitted model

Experimental data

Restoring force (N)

Relative displacement
Finite Element Modeling

Linear main structure = fairly easy to model numerically.

Nonlinear component = difficult to model numerically.
Remember

Acceleration (m/s²)

Sweep frequency (Hz)

1 g base
0.1 g base
Nonlinear Modal Analysis

Objective: investigate the nonlinear resonances.

\[ M\ddot{q}(t) + Kq(t) + f_{NL}[q(t)] = 0 \]

NNM: periodic motion (necessarily synchronously).
Nonlinear Mode #6 at Low Energy Level

Energy (J)

Frequency (Hz)

A

B

MEASURE

IDENTIFY

MODEL

UNDERSTD UNCOVER

DESIGN
Nonlinear Mode #6 at Higher Energy Level

![Graph showing frequency vs. energy (J)](image)

- **Energy (J)**
- **Frequency (Hz)**

- **MEASURE**
- **IDENTIFY**
- **MODEL**
- **UNDERSTD UNCOVER**
- **DESIGN**

Points labeled C and D on the graph, with frequency ratios 2:1, 3:1, 9:1, and 26:1 indicated.
Understand: 6\textsuperscript{th} Nonlinear Mode of the Satellite

NNM6: local deformation at low energy level

NNM6: global deformation at a higher energy level (on the modal interaction branch)

The top floor vibrates two times faster!
Troubleshooting: Problem Solved!

Frequency (Hz)

Energy (J)

2:1 interaction with NNM12
Uncovering Quasiperiodicity using Bifurcations

Inertia wheel displacement (–)

Sine-sweep at 168 N

Sweep frequency (Hz)
Eliminating Quasiperiodicity by Modifying Damping

Inertia wheel amplitude (\( \pm \))

Elastomer damping coefficient (Ns/m)

Nominal value

Annihilation of the NS bifurcations
Confirmation of the Elimination of QP

Inertia wheel displacement (–)

Sweep frequency (Hz)

Nominal design

Improved design

MEASURE
IDENTIFY
MODEL
UNDERSTD UNCOVER
DESIGN
Concluding Remarks

This is the *uninformed/biased* view of a mechanical engineer

<table>
<thead>
<tr>
<th>EE/CONTROL</th>
<th>MECHANICS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very solid theoretical framework (MLE, prediction-error ID, subspace ID)</td>
<td>Toolbox philosophy with ad hoc methods</td>
</tr>
<tr>
<td>Essentially black box</td>
<td>Often white box</td>
</tr>
<tr>
<td>Explain the data</td>
<td>Explain the system</td>
</tr>
<tr>
<td>Identification for control</td>
<td>Identification for design</td>
</tr>
<tr>
<td></td>
<td>Importance of features (modes and FRFs)</td>
</tr>
</tbody>
</table>

This is the uninformed/biased view of a mechanical engineer.
Further Points for Discussion

Choice of excitation signal (sine sweep vs. periodic random).

Noise analysis and uncertainty bounds.

Friction is challenging mechanical engineers.
Thank you for your attention!

Gaëtan Kerschen

Space Structures and Systems Laboratory
Aerospace and Mechanical Eng. Dept.
University of Liège

Colleagues and Collaborators:
J.P. Noël, J. Schoukens, K. Worden, B. Peeters