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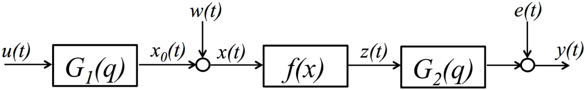
Maximum Likelihood identification of Wiener-Hammerstein model in presence of process noise

Outline

Introduction

The Wiener-Hammerstein model in presence of process noise



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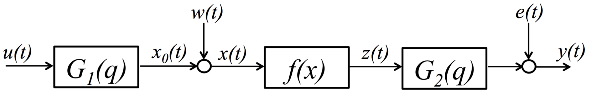

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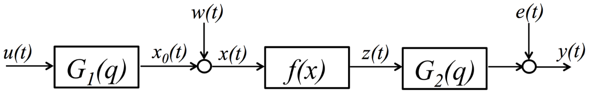

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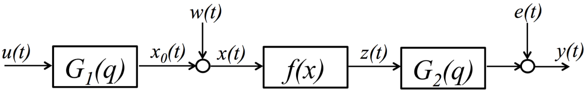

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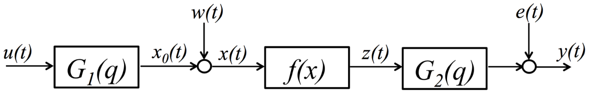

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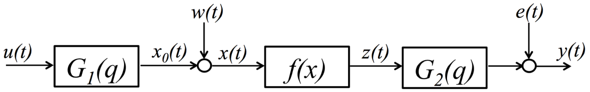

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Question: Can we use BLA in presence of process noise? How can we implement the split?

The Best Linear Approximation in case of process noise

Data generated from a Wiener-Hammerstein system

$$y(t) = G_2^0(q)f(x(t)) + e(t) \tag{1a}$$

$$x(t) = x_0(t) + w(t) \tag{1b}$$

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Theorem (Consistency of the BLA)

Signals $u(t)$, $w(t)$, $e(t)$ independent, Gaussian, stationary processes. $G(q,\theta)$ arbitrary transfer function parametrization, with freely adjustable gain. Parameter θ is estimated from u and y using an output error method,

$$\hat{\theta}_N = \underset{\theta}{argmin} \quad \frac{1}{N}\sum_{t=1}^{N}(y(t) - G(q,\theta)u(t))^2, \tag{2}$$

then

$$G(q,\hat{\theta}_N) \rightarrow kG_1^0(q)G_2^0(q) \quad w.p.\ 1 \quad as \quad N \rightarrow \infty \tag{3}$$

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- If α_i, β_i is a possible split of the dynamics in the BLA, PEM estimate is

$$\gamma* = \underset{\gamma}{\text{argmin}} \quad \frac{1}{N}\sum_{t=1}^{N}(y(t) - G_2(q,\beta_i)f(G_1(q,\alpha_i)u(t),\gamma))^2. \qquad (4)$$

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Consistency of (4) needed both for the **estimation of non-linearity** and for the **splitting algorithm.**

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- Even assuming the true split of the dynamics is known, the estimation of the non-linearity will be biased.
- In the exhaustive search algorithm, a wrong split could provide lowest prediction error. The splitting fails.

First order systems with polynomial non-linearity

Data points (1000) generated from the *true* system:

$$\begin{aligned} x_0(t) =& \frac{q}{q-a_1}u(t) \\ z(t) =& f(x_0(t)+w(t)) \\ y(t) =& \frac{q}{q-a_2}z(t)+e(t) \end{aligned} \tag{5}$$

$f(x)$ is a third degree polynomial:

$$f(x(t)) = c_0 + c_1x(t) + c_2x^2(t) + c_3x^3(t) \tag{6}$$

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- Input u is white Gaussian noise with standard deviation 1
- Process noise w is white Gaussian with standard deviation 4
- Signals u, w and e are mutually independent

First order systems with polynomial non-linearity

Monte-Carlo simulations used to generate estimation distributions over 1000 sets for

- estimation of the BLA with process noise

Table: BLA estimation

Poles/Zeros	True	Estimated $(\mu \pm \sigma)$
p_1	0.4	0.3988 ± 0.0100
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- Splitting algorithm, 2 poles $\rightarrow$ 2 combinations

Table: Splitting

	True split (RMSE)	Wrong split (RMSE)
w/out pc	**1.9474**	1.96089
w/ pc	8.11246	**8.07622**

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Monte-Carlo simulations used to generate estimation distributions over 1000 sets for

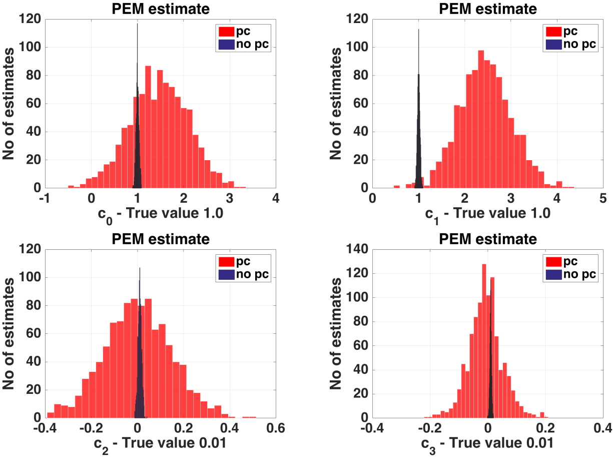
- estimation of the non-linearity with (w/) and without (w/out) process noise, for the true split

Table: Nonlinearity estimation

Par	True	Est w/ pc ($\mu \pm \sigma$)	Est w/out pc ($\mu \pm \sigma$)
c_0	1.0	1.4591 ± 0.3934	1.0010 ± 0.0009
c_1	1.0	2.4337 ± 0.3344	0.9987 ± 0.0008
c_2	0.01	0.0014 ± 0.0213	$0.0095 \pm 6.3 * 10^{-5}$
c_3	0.01	-0.0075 ± 0.0034	$0.0101 \pm 9.5 * 10^{-6}$

First order systems with polynomial non-linearity

Histograms of the estimates in presence of process noise. Parameters c_0 and c_1 show bias in the estimate.



Maximum likelihood in presence of process noise

- Main reason for inconsistency: PEM $\neq$ ML, in presence of process noise.

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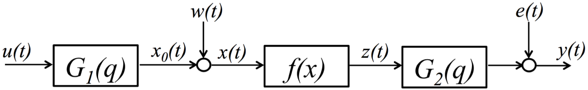

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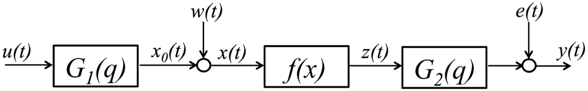

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e and w are normally distributed with zero mean and variances λ_e and λ_w respectively.

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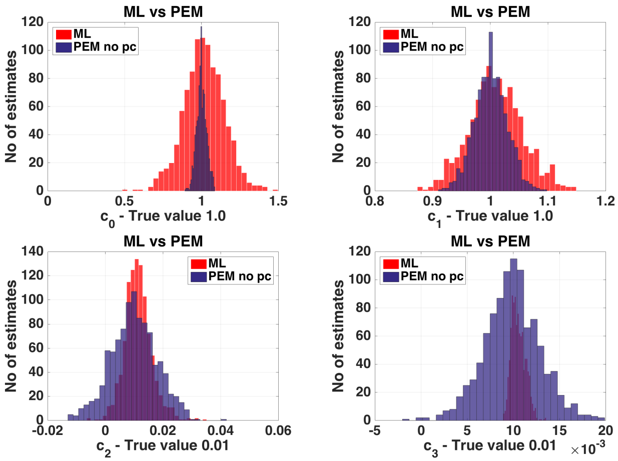
- Variances λ_e and λ_w can be also estimated.
- Numerical integration of the integral needed (e.g. Monte-Carlo integration, Importance Sampling)

Illustrative example

Fixed linear parts. Maximum likelihood estimation with process noise compared to PEM estimate with no process noise.



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- For all possible pole/zero combination the nonlinearity has been estimated using ML
- For the best split the following RMSE are obtained

Table: RMSE

Input	BLA	WHwPEM	WHwML
Multisine	0.035	0.0291	0.0162

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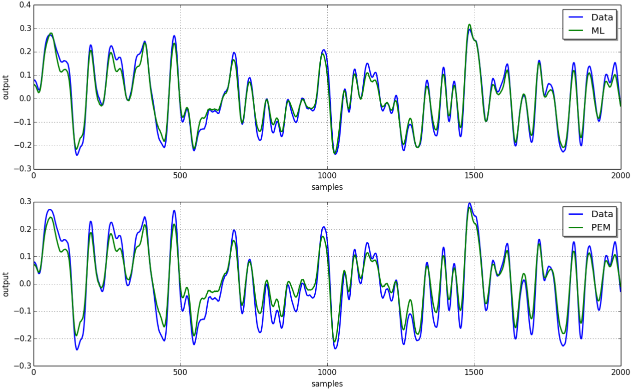

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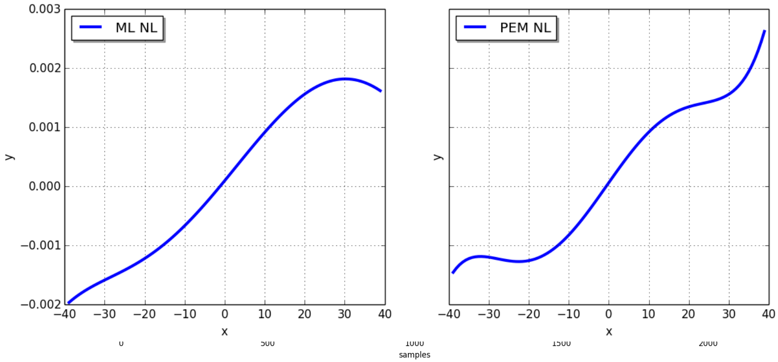

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- For colored process noise, include the noise model in the Likelihood function