Polynomial nonlinear state-space modeling of the F-16 aircraft benchmark

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Introduction: F16 ground vibration test
Goal: Capture system dynamics

\[ \text{System} \]

Inputs \rightarrow \text{System} \rightarrow \text{Outputs}
Goal: Capture system dynamics
Goal: Capture system dynamics
Set-up

Nonparametric analysis

Parametric modeling
Two inputs and three outputs are provided in the benchmark

Two inputs:
- reference input: Voltage
- actual input: Force

Three outputs:
- Acceleration at excitation location
- Acceleration on right wing
- Acceleration on payload
Multisine excitation with random frequency grid

\[ u(t) = \sum_{k=1}^{F} A_k \cos(2\pi k f_0 t + \phi_k) \]

3 amplitude levels
(12.2, 49.0, and 97.1 N RMS)

3 periods per amplitude level
(two in steady state)

10 input realizations per level
(9 for estimation, 1 for testing)

16384 points per period
Set-up

Nonparametric analysis

Parametric modeling
The FRFs and the distortion levels are estimated.
The FRFs and the distortion levels are estimated.
Focus on frequency band [4.7, 11] Hz

Output 1

Output 2

Output 3
Amplitude level 2 has the largest nonlinear distortion to signal ratio.
The nonlinear distortions at 7.3 Hz are about 25 dB larger than the noise.
Set-up

Nonparametric analysis

Parametric modeling
A linear state-space model captures dynamic behavior

\[
\begin{align*}
x(t + 1) &= A x(t) + B u(t) \\
y(t) &= C x(t) + D u(t)
\end{align*}
\]

linear state-space model
A polynomial nonlinear state-space model captures nonlinear dynamic behavior

\[
x(t + 1) = A x(t) + B u(t) + E \zeta(x(t), u(t))
\]

\[
y(t) = C x(t) + D u(t) + F \eta(x(t), u(t))
\]

linear state-space model

polynomials in \( x \) and \( u \)

with e.g. \( \zeta(x, u) = \begin{bmatrix} x_1^2 \\ x_1 x_2 \\ x_1 u \\ \vdots \\ x_2^2 u \\ u^3 \\ \vdots \end{bmatrix} \)
Identification of a polynomial nonlinear state-space model

\[
x(t + 1) = A x(t) + B u(t) + E \zeta(x(t), u(t))
\]

\[
y(t) = C x(t) + D u(t) + F \eta(x(t), u(t))
\]

linear state-space model

polynomials in \( x \) and \( u \)

\[
\theta = \begin{bmatrix}
  \text{vec}(A) \\
  \text{vec}(B) \\
  \text{vec}(C) \\
  \text{vec}(D) \\
  \text{vec}(E) \\
  \text{vec}(F) \\
  \text{vec}(x(0)) \\
  \text{vec}(u(0))
\end{bmatrix}
\]

\[
\epsilon(k, \theta) = Y(k, \theta) - Y_{\text{meas}}(k)
\]

\[
K_{\text{WLS}}(\theta) = \sum_{k=1}^{N_F} \epsilon^H(k, \theta) W(k) \epsilon(k, \theta)
\]

\[
\hat{\theta} = \arg \min_{\theta} K_{\text{WLS}}
\]
Nonlinear interaction only within first resonance and coupled interaction in other three resonances.
Conclusions and future work

Nonparametric analysis:
  High-quality measurements
  Some room for improvement with nonlinear modeling

Parametric modeling:
  Frequency weighting possible
  Challenging benchmark

Future work:
  Non-polynomial basis functions
Two inputs and three outputs are provided in the benchmark:

**Two inputs:**
- reference input: Voltage
- actual input: Force

**Three outputs:**
- Acceleration at excitation location
- Acceleration on right wing
- Acceleration on payload

**Sampling frequency:**
- 400 Hz
  (upsampled from 200 Hz)
Amplitude level 2 has the largest nonlinear distortion to signal ratio.
The nonlinear distortions at 7.3 Hz are about 25 dB larger than the noise.
Focus on main resonance with second-order linear and nonlinear model (lowest amplitude)

![Graphs showing output, linear error, and PNLSS error for three different outputs.](image)

- Output 1
  - Frequency (Hz): 7.3, 8.6
  - Amplitude (dB): 59, 47, 21
- Output 2
  - Frequency (Hz): 7.3, 8.6
  - Amplitude (dB): 65, 53, 27
- Output 3
  - Frequency (Hz): 7.3, 8.6
  - Amplitude (dB): 64, 53, 27

PNLSS error

95 parameters
Focus on main resonance with second-order linear and nonlinear model (amplitude level 2)
Limit the number of parameters by allowing nonlinear interaction only within a resonance

\[
\begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3 \\
  x_4 \\
  x_5 \\
  x_6 \\
  x_7 \\
  x_8 \\
  x_9 \\
  x_{10} \\
  x_{11} \\
  x_{12}
\end{bmatrix}^+ = \begin{bmatrix}
  A_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & A_2 & 0 & 0 & 0 & 0 \\
  0 & 0 & A_3 & 0 & 0 & 0 \\
  0 & 0 & 0 & A_4 & 0 & 0 \\
  0 & 0 & 0 & 0 & A_5 & 0 \\
  0 & 0 & 0 & 0 & 0 & A_6
\end{bmatrix} x(t) + Bu(t) + \\
\begin{bmatrix}
  f_1(x_1, x_2) \\
  f_2(x_1, x_2) \\
  f_3(x_3, x_4) \\
  f_4(x_3, x_4) \\
  f_5(x_5, x_6) \\
  f_6(x_5, x_6) \\
  f_7(x_7, x_8) \\
  f_8(x_7, x_8) \\
  f_9(x_9, x_{10}) \\
  f_{10}(x_9, x_{10}) \\
  f_{11}(x_{11}, x_{12}) \\
  f_{12}(x_{11}, x_{12})
\end{bmatrix}
\]

\[
y(t) = Cx(t) + Du(t) + g_1(x_1, x_2) + g_2(x_3, x_4) + g_3(x_5, x_6) + g_4(x_7, x_8) + g_5(x_9, x_{10}) + g_6(x_{11}, x_{12})
\]
Limit the number of parameters by allowing nonlinear interaction only within a resonance

\[
\begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3 \\
  x_4 \\
  x_5 \\
  x_6 \\
  x_7 \\
  x_8 \\
  x_9 \\
  x_{10} \\
  x_{11} \\
  x_{12}
\end{bmatrix} +
\begin{bmatrix}
  A_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & A_2 & 0 & 0 & 0 & 0 \\
  0 & 0 & A_3 & 0 & 0 & 0 \\
  0 & 0 & 0 & A_4 & 0 & 0 \\
  0 & 0 & 0 & 0 & A_5 & 0 \\
  0 & 0 & 0 & 0 & 0 & A_6
\end{bmatrix} x(t) + Bu(t) +
\begin{bmatrix}
  f_1(x_1, x_2) \\
  f_2(x_1, x_2) \\
  f_3(x_3, x_4) \\
  f_4(x_3, x_4) \\
  f_5(x_5, x_6) \\
  f_6(x_5, x_6) \\
  f_7(x_7, x_8) \\
  f_8(x_7, x_8) \\
  f_9(x_9, x_{10}) \\
  f_{10}(x_9, x_{10}) \\
  f_{11}(x_{11}, x_{12}) \\
  f_{12}(x_{11}, x_{12})
\end{bmatrix}
\]

\[
y(t) = Cx(t) + Du(t) + g_1(x_1, x_2) + g_2(x_3, x_4) + g_3(x_5, x_6)
\]
\[
+ g_4(x_7, x_8) + g_5(x_9, x_{10}) + g_6(x_{11}, x_{12})
\]
Nonlinear interaction only within a resonance (12th order models)

350 parameters (full model would have 7826 parameters)
Nonlinear interaction only within first resonance and coupled interaction in other three resonances

Output 2

Output 3

882 parameters (full model would have 7826 parameters)