

Regressor selection using Lipschitz quotients on the F-16 aircraft benchmark



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ARRS L2-8174 »Method for the
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0001



Lipschitz quotients





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- Lipschitz continuity: $|\partial f / \partial x_i|$ bounded
 q_{ij} bounded



Lipschitz quotients

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- Proposed in He & Asada (1993) for identifying system orders
- Used for identifying regressors: MATLAB `sequentialfs()`
 - (sequential feature selection)
 - backward & forward & backward until stabilized



Example





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J.P. Noël, M. Schoukens, *F-16 aircraft benchmark based on ground vibration test data*



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- 14742 regressor vectors analysed
 - 40 components
 - 20 delayed inputs, 20 delayed outputs

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- Regressor selection: 13 regressors selected in 10565 s

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 - squared exponential kernel (a priori with hyperparameters), zero mean, and noise with unknown variance
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- Same prediction but with all 40 regressors: $e_{\text{RMSt}}=0.0158$, calculated in 5.62 seconds (after 1123 s of optimization)



Comparison

- How do models based on 13 favourite regressors of the other selection methods perform?



Comparison

Method	e_{RMSt}	Time for selection [s]
Lipschitz	0.0303	10565
CCorr	0.0221	<1
dCorr	0.0221	272
PCorr	0.0160	6
MI	0.0218	3
PMI	0.0227	77
ANOVA	0.0191	<1
LIP (embedded)	0.0171	7560
All 40 regressors	0.0158	-

ProOpter, J. Kocijan et al., *Regressor selection for ozone prediction*, Simulation Modelling Practice and Theory **54** (2015) 101–115

