Data Driven Discrete Time Modeling of Continuous Time Nonlinear Systems

Problems, Challenges, Success Stories

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$f(y, u, \theta)$
System Identification

Data

Model

Distance
System Identification

Data

Linear System
\[ Y = GU \]

Model

Distance
Problem solved? Mission accomplished?
System Identification in a real world

Linear

Nonlinear

Time-Varying
Linear SI versus Nonlinear SI

Linear SI
- mature field
- well developed tools
- inexpensive
Linear SI versus Nonlinear SI

Linear SI
- mature field
- well developed tools
- inexpensive

Nonlinear SI
- hot research topic
- expensive
Linear SI versus Nonlinear SI

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Preliminary Questions
- Do we face a nonlinear identification problem?
- Safe to use a linear system identification approach?
- How much to gain with a nonlinear model?
Do we need a nonlinear model?
Detection, qualification, quantification NL

Linear identification in the presence of nonlinear distortions

Nonlinear system identification
Detection, qualification, quantification NL

Goal
characterize nonlinear behaviour
no increase of the measurement time
little user interaction

Result: a well informed decision: linear or nonlinear model?
Detection, qualification, quantification NL

Goal
characterize nonlinear behaviour
no increase of the measurement time
little user interaction

Tool
periodic excitations

\[ u(t) = \frac{1}{F} \sum_{k=1}^{F} A_k \cos(2\pi kf_0 t + \varphi_k) \]
Detection, Qualification, Quantification NL

Basic Idea

\[ u(t) = 2 \cos \omega t = e^{j\omega t} - e^{-j\omega t}, \ \omega = 1, \]
Detection, Qualification, Quantification NL

Basic Idea

\[ u(t) = 2 \cos \omega t = e^{j \omega t} - e^{-j \omega t}, \; \omega = 1, \]

\[ U(\omega) \]

\[ -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \]

\[ f \]

\[ u^3 = (e^{j \omega t} - e^{-j \omega t})(e^{j \omega t} - e^{-j \omega t})(e^{j \omega t} - e^{-j \omega t}) \]
Detection, Qualification, Quantification NL
Basic Idea

\[ u(t) = 2 \cos \omega t = e^{j\omega t} - e^{-j\omega t}, \ \omega = 1, \]

\[ U(\omega) \]

\[ f \]

\[ u^3 = (e^{j\omega t} - e^{-j\omega t})(e^{j\omega t} - e^{-j\omega t})(e^{j\omega t} - e^{-j\omega t}) \]

Output: all possible combinations, 3 by 3, of the frequencies -1 and 1

\[
\begin{array}{cccc}
1 & 1 & 1 & 3 \\
1 & 1 & -1 & 1 \\
1 & -1 & 1 & 1 \\
1 & -1 & -1 & -1 \\
-1 & 1 & 1 & 1 \\
-1 & 1 & -1 & -1 \\
-1 & -1 & 1 & -1 \\
-1 & -1 & -1 & -3 \\
\end{array}
\]
Detection, Qualification, Quantification NL

Basic Idea

input

linear

+ even

+ odd

= output
Detection, qualification, quantification of nonlinear distortions

\[ u + \frac{1}{ms^2 + ds + k_1} \rightarrow y \]

\[ k_3 y^3 \]
Detection, qualification, quantification of nonlinear distortions

\[ y = \frac{1}{ms^2 + ds + k_1} u + k_3 y^3 \]

odd NL

even NL

noise

![Image of a device with controls and knobs]
Detection, qualification, quantification of nonlinear distortions

Acknowledgement: These measurements were done in a collaboration with Bart Peeters (LMS International, part of Siemens Product Lifecycle Management), Jean-Philippe Noël and Gaetan Kerschen (University of Liège) during the LMS Ground Vibration Testing Master Class held in September 2014 at the Saffraanberg military basis in Belgium
Detection, qualification, quantification of nonlinear distortions

Amplitude (dB)

Frequency (Hz)

odd NL
even NL
noise
Detection, qualification, quantification of nonlinear distortions

odd NL

even NL

noise

FRF (dB)

Frequency (Hz)

Amplitude (dB)

Frequency (Hz)
Do we need a nonlinear model?  
Detection, qualification, quantification NL

Linear identification in the presence of nonlinear distortions

Nonlinear system identification
Nonlinear system identification

Data

Model

Distance
Nonlinear system identification

Model
- Continuous time - Discrete time
- Unstructured - Structured
- Coupled - Decoupled

Data
- Reflect future use
- Input design

Cost
- Model errors - Noise errors
- Process noise

Success stories
Model: Continuous time - Discrete time
ZOH-Assumption

Physical system

\[
\frac{dx(t)}{dt} = F_{CT}(x(t), u(t))
\]

What we like: explicit model

\[
x_{DT}(k+1) = F_{DT}(x_{DT}(k), u_{DT}(k))
\]

What we get: implicit model

\[
x_{DT}(k) = \tilde{F}_{DT}(x_{DT}(k), u_{DT}(k))
\]

Goodwin et. al. (2013): Truncated Taylor Series to get explicit model

\[
x_{DT}(k) = x_{CT}(kT_s) + O(T_s)
\]

Assumption: NLSS is affine in the input
Model: Continuous time - Discrete time
Alternative Assumption: Low Pass Assumption

Bandlimited signal

\[ u(t) = L(u_{t-1}) + 0 \]

Low pass signal

\[ u(t) = L(u_{t-1}) + O\left(\left(\frac{B}{f_S}\right)^{-(d_u - 0.5)}\right) \]
Experimental verification on silverbox

Output Spectrum

Relative Error

30 dB/decade
Nonlinear models - User desires
Unstructured - Structured models

A model should be more than a bunch of parameters

Flexible: cover a wide range of nonlinear systems
Multiple Input - Multiple Output
Reliable
Provides intuitive insight in system behaviour
Nonlinear Models

**unstructured**

\[ x_+ = Ax + Bu + F(x, u) \]

**structured**

\[ y = Cx + Du + G(x, u) \]

- **Hammerstein**
  - Parallel Hammerstein
  - Hammerstein-Wiener
  - Parallel Hammerstein-Wiener
- **Wiener**
  - Parallel Wiener
  - Wiener-Hammerstein
  - Parallel Wiener-Hammerstein
- **Wiener-Hammerstein Feedback**
- **Nonlinear LFR Feedback**
Nonlinear Models

unstructured

\[ x_+ = Ax + Bu + F(x, u) \]
\[ y = Cx + Du + G(x, u) \]
Nonlinear Models

unstructured

\[ x_+ = Ax + Bu + F(x, u) \]
\[ y = Cx + Du + G(x, u) \]

NN, polynomials, ...

System identification: Best Linear Approximation
Nonlinear Models

should be more than a bunch of numbers

\[ x_+ = Ax + Bu + F(x, u) \]
\[ y = Cx + Du + G(x, u) \]

\[ F_1(x, u) = a_1 x_1 + a_2 x_2 + a_3 x_3 \]
\[ a_1 x_1^2 + a_{12} x_1 x_2 + a_{13} x_1 x_3 + a_{22} x_2^2 + a_{12} x_2 x_3 + a_{33} x_3^2 \]
\[ a_{111} x_1^3 + a_{112} x_1 x_1 x_2 + a_{113} x_1 x_1 x_3 + a_{123} x_1 x_2 x_3 + a_{133} x_1 x_3 x_3 \]
\[ a_{222} x_2^3 + a_{212} x_2 x_1 x_2 + a_{232} x_2 x_3 x_2 + a_{233} x_2 x_3 x_3 + \ldots \]
Decoupling multivariate polynomials

- increased physical insight
- number of parameters grows linearly instead of combinatorially
- exact - approximatively - noise weighting

- in NN are all $g_i$ are the same
- NN are universal approximators, rate $1/\sqrt{n}$
Decoupling nonlinear state space models

\[ x(k + 1) = Ax(k) + Bu(k) + F(x(k), u(k)) \]
\[ y(k) = Cx(k) + Du(k) + H(x(k), u(k)) \]

1) full multivariate description \( F \)

2) Semi-decoupling: all coupling terms eliminated \( F \rightarrow G \)
   - each \( G_i(z_1, \ldots, z_n) = \sum_{j=1}^{n} g_{ij}(z_j) \) with \( i = 1, \ldots, n \)
   - number of terms \( n^2d \) proportional with degree \( d \) \( \gg \) combinatorial
   - can be calculated from the decoupling matrices

3) full decoupling
   each state depends nonlinearly only on one other state
   - is this (always) possible?
Example

Low order coupled model

Low order decoupled model

High order decoupled model

Output spectrum

Static nonlinearity estimate

Output (V)

Input (V)

-4 -2 0 2 4
Nonlinear system identification

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Cost
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- Process noise
  Joint $\bowtie$ Marginal distribution

Success stories
Data: reflect future use: wet clutch example

Cross-sectional schematic of the electro-hydraulic valve and wet-clutch.

Fig. 3. (Top) Normally distributed multisine signal and (bottom) amplitude distribution.

Fig. 4. (Top) Positively skewed multisine signal and (bottom) amplitude distribution.

Fig. 5. Bandlimited filling signal with a superimposed multisine.

Fig. 9. Amplitude spectrum of the filling signal as a combination of two spectra. Crosses: bandlimited approximate spectrum and circles: superimposed multisine spectrum.

Wet Clutch results

Cross-sectional schematic of the electro-hydraulic valve and wet-clutch.
Experiment design for nonlinear systems
Optimal input design

Not solved

Not understood

What we know
- power spectrum
- amplitude distribution
- global solution --> a combined optimization is needed

Simple example (M. Gevers)

\[ y(t) = [b_1 u(t) + b_2 u(t - 1)]^2 + v(t) \]

\[ p(u(t), u(t - 1)) \]

\[ \text{Optimize joint distribution} \]

\[ F = \begin{bmatrix} I_{11} & I_{12} \\ I_{21} & I_{22} \end{bmatrix} \]

\[ I_{11} = \frac{4}{\sigma_v^2} \left( b_1^2 m_u^{10} + 2b_1 b_2 m_u^{21} + b_2^2 m_u^{22} \right) \]

\[ I_{12} = I_{21} = \frac{4}{\sigma_v^2} \left( b_1^2 m_u^{31} + 2b_1 b_2 m_u^{22} + b_2^2 m_u^{13} \right) \]

\[ I_{22} = \frac{4}{\sigma_v^2} \left( b_1^2 m_u^{22} + 2b_1 b_2 m_u^{13} + b_2^2 m_u^{04} \right) \]

Optimal input design

What we are not able to do
- what is the global solution?
- how to generate a random signal with an arbitrary joint distribution

We look into sub-optimal solutions
- Gaussian mixtures
- Dictionary of excitation signals
- Brute force optimization of a single realization
Nonlinear system identification

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Success stories
Choice of the cost function in the presence of model errors

Some thoughts on model errors

Avoidable model errors

  e.g.: unmodelled dynamics in linear SI
  Why wouldn’t you eliminate these?

Non-avoidable model errors

  e.g.: NL system is outside the NL model class
  User defined criterion needed: how to shape the error?
  Shaping with the noise variance is no obvious choice
Cost: Model errors - Noise errors
noise weighting $\leftrightarrow$ shaping the model errors

Typical result nonlinear identification

The errors are not dominated by the noise

The error weighting in the cost function becomes a user choice:

e.g. small error in freq. band of interest

Using the noise weighting can destroy the quality of the fit

Cost: Process noise

Wiener system example

\[ x_0(t) = G(q, \theta)u(t) \]
\[ x(t) = x_0(t) + w(t) \]
\[ y(t) = f(x(t), \eta) + e(t) \]

- noise \( w(t) \) and signal \( u(t) \) get mixed in \( f(\cdot) \)
- neglecting process noise can lead to an arbitrary large bias
- solving the full problem is much more involved
  \[ \rightarrow \text{ tools needed to detect the presence of process noise} \]

Nonlinear system identification

Model
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- Reflect future use
- Input design
  Input domain?
  Optimal input design

Cost
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Success stories
Nonlinear identification of a battery

Lithium Ion Polymer Battery (EiG-ePLB-C020, Li(NiCoMn))

3.65 V, 20 Ah, AC impedance (1 kHz < 3 mΩ)

Nonparametric distortion analysis
Parametric NLSS model results

No. of States = 4
Degree of Nonlinearity = upto degree 3

Time domain

Frequency domain
Conclusions

Do we need a nonlinear model?

Linear identification in the presence of nonlinear distortions

Nonlinear System Identification

Model
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Data
  - Reflect future use
  - Input design

Input domain?

Optimal input design

Cost
  - Model errors - Noise errors
  - Process noise

Success stories